Stable Electrostatic Wakefield in Quantum Nanowires

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❖ Quantum Plasma:

- Composed of electrons, ions and is characterized by high particle density & low temperature.
- Schrödinger-Poisson & Wigner-Poisson models
- Quantum hydrodynamical (QHD) model deals with the quantum corrections in plasmas.

❖ Existence of Quantum Plasmas:

- Metallic nanostructures $(n \sim 10^{21} - 10^{22}\,\mathrm{cm}^{-3})$ (metal clusters, nanoparticles, and thin metal film)
- Semiconductor materials $(n \sim 10^{17} - 10^{19}\,\mathrm{cm}^{-3})$
- Dense astrophysical objects $(n \sim 10^{30}\,\mathrm{cm}^{-3})$

❖ Quantum Scales

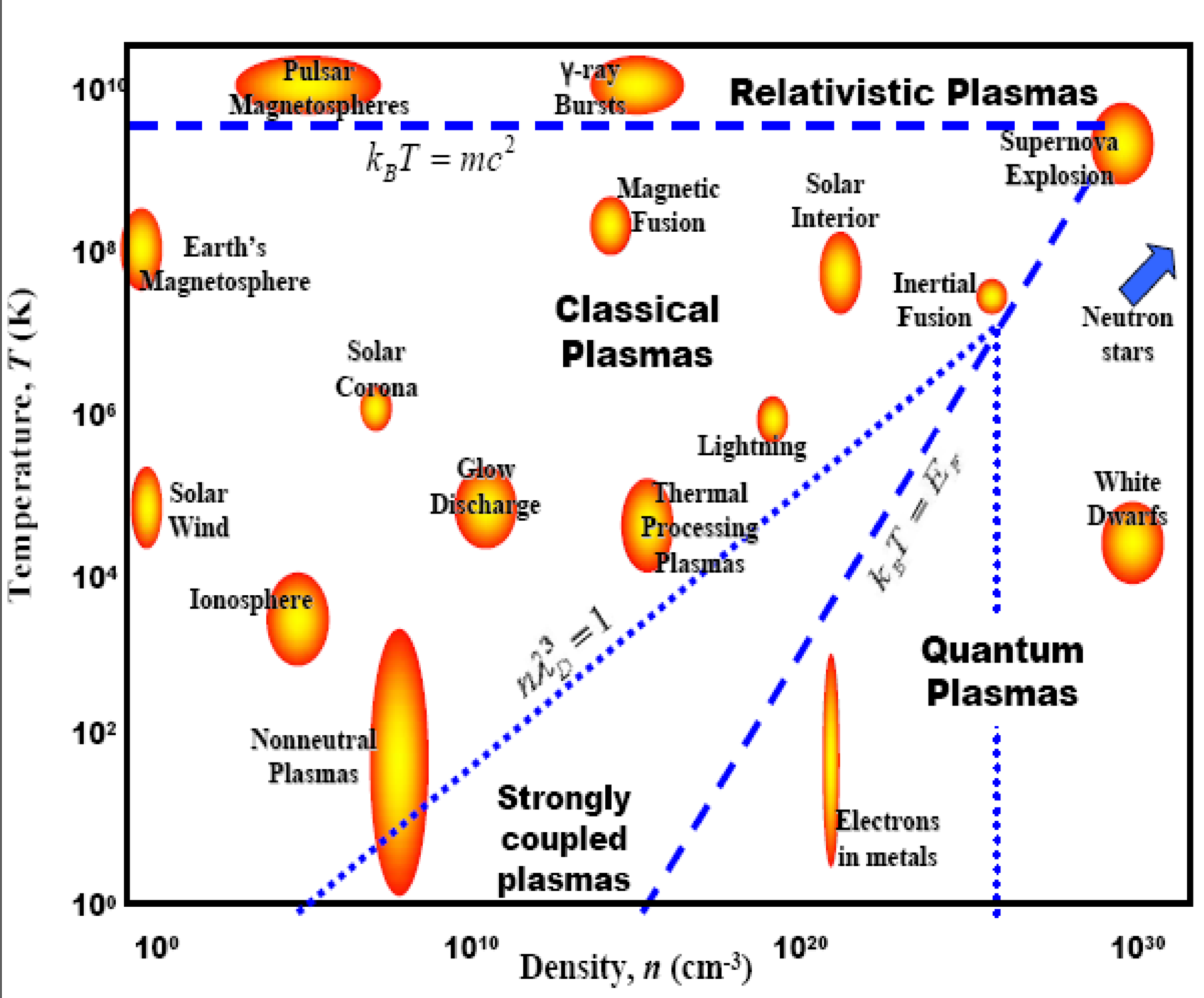
- Plasma frequency $\omega_P = \left(\frac{n_0 e^2}{\varepsilon_0 m_e}\right)^{1/2}$
- Fermi velocity $V_F = \left(\frac{2E_F}{m_e}\right)^{1/2}$
- Fermi energy $E_F = E_{kin} = \left(\frac{\hbar^2}{2m_e}\right)(3\pi^2 n_0)^{2/3}$
- Quantum coupling parameter $\Gamma_Q = \frac{E_{int}}{E_{kin}} \sim \left(\frac{1}{n_0\lambda_F^3}\right)^{2/3} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2$
- Quantum analog of Debye length

$$\lambda_F = V_F/\omega_p \qquad \lambda_F \to \lambda_D,\ \Gamma_Q \to \Gamma_C$$

- Quantum limits & conditions

$$n_0\lambda_B^3 \geq 1 \qquad \lambda_B \sim \lambda_F > \lambda_D \qquad E_F > k_B T$$

Plasma Physics diagram



❖ Model:

- Excitation of the wakefield due to an electron beam
- A nanowire which is composed of electrons & ions with a radius "a"
- Immersed in a strong external magnetic field $\mathbf{B} = \hat{\mathbf{z}}B_0$
- The electron plasma wave (EPW), in which the electrons are mobile while ions are assumed static in the background plasma.
- The equilibrium charge-neutrality condition

$$n_{e0} \approx n_{i0} \approx n_0$$

❖ QHD Equations:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{U}) = 0, \qquad (1)$$

$$m_e n\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)\mathbf{U} = e\left(\nabla\phi - \frac{\mathbf{U}\times\mathbf{B}}{c}\right) - \nabla P_F - \nabla\phi_B, \qquad (2)$$

$$\nabla^2\phi = \frac{e}{\varepsilon_0}(n + N - n_{i0}), \qquad (3)$$

where the Fermi pressure & quantum Bohm potential are, respectively, given by

$$P_F = \frac{m_e V_F^2}{3n_0^2} n^3, \qquad \phi_B = \frac{-\hbar^2}{2m_e\sqrt{n}}\nabla^2\sqrt{n}$$

❖ Trivelpiece Gould Configuration:

- Decompose Laplacian operator into transverse and longitudinal components $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial_z^2$.
- Transform the perturbations in terms of Bessel function

$$\Psi(r,\theta,z,t) = \sum_{l,m=0}^{\infty}\Psi_{l,m}(z,t)J_m(k_{\perp l,m}r)\exp(im\theta), \qquad (4)$$

where $\Psi(r,\theta,z,t) = (n,\phi,U,N)$,

$\Psi_{l,m}(z,t)$ the arbitrary amplitudes of the perturbations,

$k_\perp (= \alpha_{l,m}/a)$ the perpendicular component and

$\alpha_{l,m}$ the lth zero of the Bessel function of the order m.

❖ Wakefield:

Using Eq. (4), the linearized set of Eqs. (1)-(3) becomes

$$\frac{\partial n_{l,m}}{\partial t} + n_0\frac{\partial U_{l,m}}{\partial z} = 0, \qquad (5)$$

$$\frac{\partial U_{l,m}}{\partial t} = \frac{e}{m_e}\frac{\partial\phi_{l,m}}{\partial z} - \frac{V_F^2}{n_0}\frac{\partial n_{l,m}}{\partial z} + \frac{\hbar^2}{4m_e^2 n_0}\frac{\partial^3 n_{l,m}}{\partial z^3}, \qquad (6)$$

$$\left(\frac{\partial^2}{\partial_z^2} - k_{\perp l,m}^2\right)\phi_{l,m} = \frac{e}{\varepsilon_0}(n_{l,m} + N_{l,m}) \qquad (7)$$

Combining Eqs. (5)-(7), we obtain

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 - V_F^2\frac{\partial^2}{\partial z^2} + \frac{\hbar^2}{4m_e^2}\frac{\partial^4}{\partial z^4}\right)\left(\frac{\partial^2}{\partial z^2} - k_{\perp l,m}^2\right)n_{l,m}$$
$$+ k_{\perp l,m}^2\omega_p^2 n_{l,m} = -\omega_p^2\frac{\partial^2}{\partial z^2}N_{l,m} \qquad (8)$$

$$\omega_{l,m}^2 = \frac{\omega_p^2 k^2}{k^2 + k_{\perp l,m}^2} + V_F^2 k^2 + \frac{\hbar^2}{4m_e^2}k^4. \qquad (9)$$

Using $\zeta = z - V_0 t$ and $\tau = t$, Eq. (8) yields

$$\left(\frac{\partial^2}{\partial\tilde{\tau}^2} + (\tilde{V}_0^2 - 1)\frac{\partial^2}{\partial\tilde{\zeta}^2} - 2\tilde{V}_0\frac{\partial^2}{\partial\tilde{\tau}\partial\tilde{\zeta}} + 1 + \Lambda^4\frac{\partial^4}{\partial\tilde{\zeta}^4}\right)$$
$$\left(\frac{\partial^2}{\partial\tilde{\zeta}^2} - \tilde{k}_{\perp l,m}^2\right)\tilde{n}_{l,m} + \tilde{k}_{\perp l,m}^2\tilde{n}_{l,m} = -\frac{\partial^2}{\partial\tilde{\zeta}^2}\tilde{N}_{l,m} \qquad (10)$$

For $\partial/\partial\tau \to 0$

$$\left(\Lambda^4\frac{\partial^4}{\partial\tilde{\zeta}^4} + \tilde{k}_a^2\frac{\partial^2}{\partial\tilde{\zeta}^2} + \tilde{k}_b^4\right)\tilde{n}_{l,m} = -\tilde{N}_{l,m} \qquad (11)$$

where $\Lambda^4 = \lambda_q^4/\lambda_F^4$,

$$k_a^2 = \tilde{V}_0^2 - 1 - \Lambda^4\tilde{k}_{\perp l,m}^2,$$

$$k_b^4 = 1 + \tilde{k}_{\perp l,m}^2(1 - \tilde{V}_0^2).$$

A normalized solution of (11) can be expressed, as

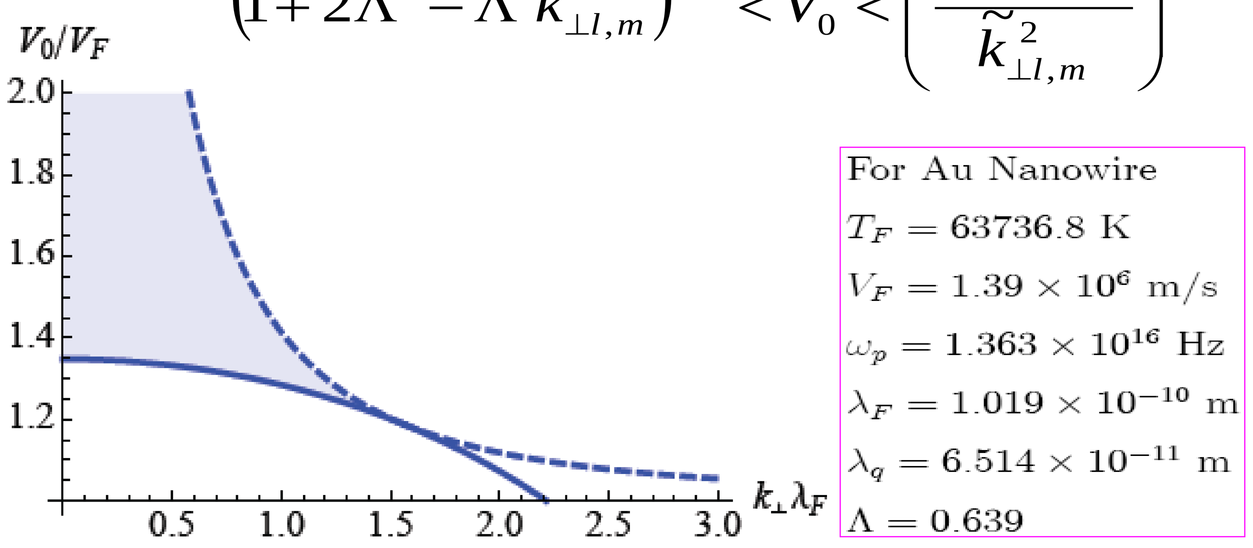
$$n_{l,m}(\zeta) = \frac{1}{k_+^2 - k_-^2}\int_{-\infty}^{\infty}d\zeta_0\left(\frac{\sin k_+(\zeta - \zeta_0)}{k_+} - \frac{\sin k_-(\zeta - \zeta_0)}{k_-}\right)$$
$$\times\Theta(\zeta_0 - \zeta)N_{l,m}(\zeta_0), \qquad (12)$$

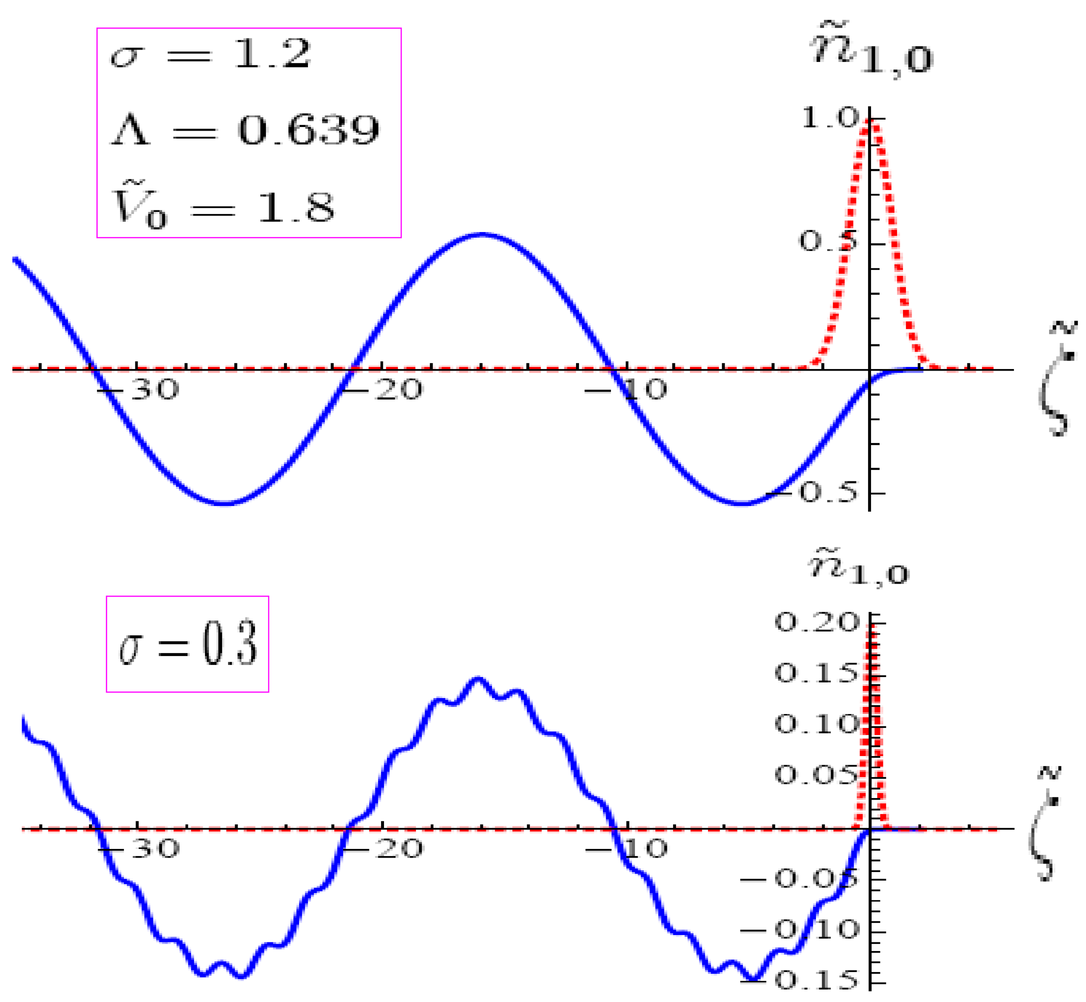
$$k_\pm^2 = \frac{k_a^2 \pm\sqrt{k_a^4 - 4\Lambda^4 k_b^4}}{2\Lambda^4}, \qquad N_{l,m}(\zeta_0) = N_0\exp\left(-\frac{\zeta_0^2}{\sigma^2}\right)$$

For inequalities: $k_a^2 > 0$, $k_b^4 > 0$,

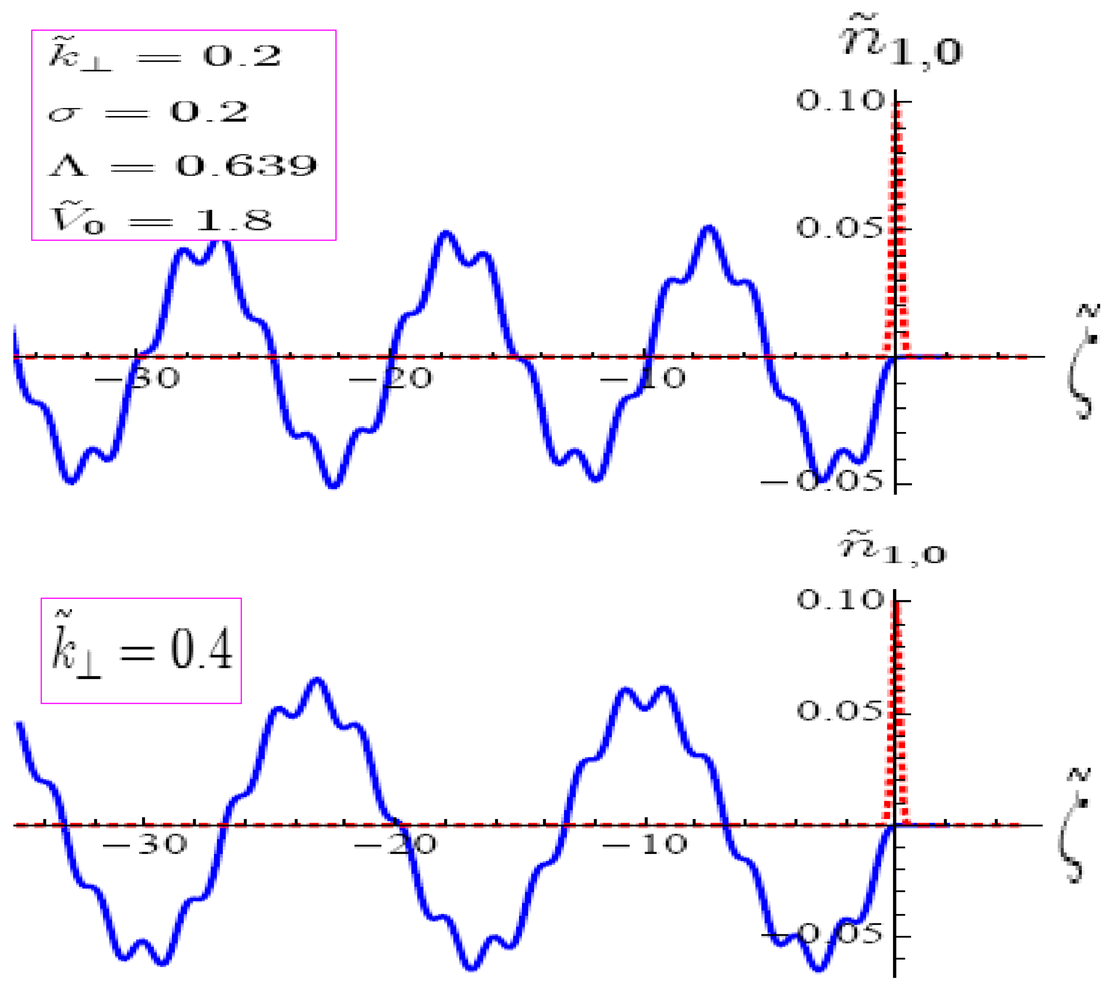
$k_a^4 > 4\Lambda^4 k_b^4$, we obtain

$$\left(1 + 2\Lambda^2 - \Lambda^4\tilde{k}_{\perp l,m}^2\right)^{1/2} < \tilde{V}_0 < \left(\frac{1 + \tilde{k}_{\perp l,m}^2}{\tilde{k}_{\perp l,m}^2}\right)^{1/2}$$





Variation of the width of an electron pulse



Variation of the wavenumber

❖ Conclusions:

- Quantum force introduces an additional effect to the EPW dispersion besides the statistical pressure effect.
- We have derived an expression for the EPW wakefield excited by an electron pulse obtaining its conditions of existence in a dense quantum Au nanowire.
- Quantum effects appear on the EPW wakefield for varying width through the condition $k_+\sigma \lesssim 1$
- The amplitudes of the EPW wakefield become pronounced for different wavenumbers.
- The study will be helpful for particle acceleration.