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Kinetic full wave analysis of Bernstein waves in tokamak plasmas

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Introduction and Motivation

- Externally launched electromagnetic waves (EM) seem to be the best choice for **microwave plasma heating and non-inductive current drive** in in high-density core plasmas of spherical tokamaks due to possibility of **off-axis launching and highly localized heating.**
- **Bernstein waves** in spherical tokamaks owes to the electron cyclotron (EC) wave mode conversion which has attracted growing attention due to its potential in heating the plasma.
- The usual analysis by **ray tracing method** is not applicable in a plasma with high density or low magnetic field becausethe **the presence of cutoff layer** may prevent the waves from penetrating into the central part from the low field side.
- In this case, **full wave analysis of EBW (Electron Bernstein Wave)** is required waves is required for evaluating the absorption profile and optimizing the wave launching conditions.
- In case of parallel motion iwith no FLR effects, **beach heating at EC resonance** takes place and full wave analysis provides sufficiently accurate method to evaluate the power absorption.
- In the present analysis, the **full wave analysis of EBW using integral formulation and beach cyclotron heating at EC resonance** in a magnetized plasma are discussed.

Full Wave Analysis

- **Solves Maxwell's equation as a boundary-value problem no simplifying assumptions, non-local fields, fixed ω**
  - $\boldsymbol{E}$: wave electric field
  - $\overleftrightarrow{\epsilon}$: dielectric tensor

$$\nabla\times\nabla\times\boldsymbol{E}=\frac{\omega^2}{c^2}\overleftrightarrow{\epsilon}\cdot\boldsymbol{E}+\mathrm{i}\,\omega\mu_0\boldsymbol{j}_{\mathrm{ext}}$$

- **Merit of full wave analysis**
  - Wave length longer than the scale length of medium
  - Propagation over an evanescent layer
  - Coupling to antenna
  - Formation of standing wave
- **Method of full wave analysis**
  - **Fourier analysis**: algebraic equation
  - **Discrete differential equation**: finite diff./element method
  - **Mixture of above two methods**

Description of FLR Effects in Full Waves Analyses

- **Fast wave approximation:**
  - Evaluate wave length of fast waves using cold plasma approx.
  - Only for fast wave and propagating waves, no standing waves
- **Differential operator**: TORIC code (Brambilla, IPP)
  - Expand with respect to $k_\perp\rho$, and replace it with $\mathrm{i}\,\rho\partial/\partial r_\perp$
  - Limited to $k_\perp\rho\lesssim 1$, and up to the second order of $k_\perp\rho$
- **Fourier transform:** AORSA code (Jaeger, ORNL)
  - FT in the direction of homogeneity, and convolution integral
  - All Fourier components are coupled, requires large computational resources
- **Integral operator**: Sauter (NF, 1992), TASK/W1 (Fukuyama)
  - Integral form of dielectric tensor: $\int\epsilon(x-x')\cdot E(x')\mathrm{d}x'$
  - Localized in space, less computational resources required.

Integral Formulation of Wave-Particle Interaction

- **Particle orbit**: $\boldsymbol{r}=\boldsymbol{r}'+\Delta\boldsymbol{r}(\boldsymbol{v},\boldsymbol{r},t-t');\quad \boldsymbol{v}=\boldsymbol{v}'+\Delta\boldsymbol{v}(\boldsymbol{v},\boldsymbol{r},t-t')$
- **Perturbed distribution from Vlasov equation**:

$$f(\boldsymbol{r},\boldsymbol{v},t)=-\frac{q}{m}\int_{-\infty}^{t}\mathrm{d}t'\left[\boldsymbol{E}(\boldsymbol{r}')+\boldsymbol{v}'\times\boldsymbol{B}(\boldsymbol{r}')\right]\cdot\frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial\boldsymbol{v}'}\,\mathrm{e}^{-\mathrm{i}\,\omega t'}$$

- **Induced current**:

$$\boldsymbol{j}(\boldsymbol{r})=\int\mathrm{d}\boldsymbol{v}\,q\boldsymbol{v}f(\boldsymbol{r},\boldsymbol{v},t)\,\mathrm{e}^{\mathrm{i}\,\omega t}=\int\mathrm{d}\boldsymbol{r}'\,\overleftrightarrow{\sigma}(\boldsymbol{r}-\boldsymbol{r}',t-t')\cdot E(\boldsymbol{r}')$$

- **The integral form of the conductivity tensor** is defined by

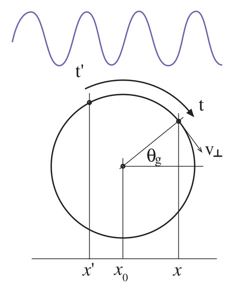
$$\overleftrightarrow{\sigma}(\boldsymbol{r},\boldsymbol{r}',t-t')=-\frac{q}{m}\int_{-\infty}^{t}\mathrm{d}t'\frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial\boldsymbol{v}'}\cdot\left[\boldsymbol{v}+\frac{1}{\mathrm{i}\,\omega}\boldsymbol{v}\cdot\boldsymbol{v}'\times\nabla\times\right]\Bigg|_{\substack{\boldsymbol{r}'=\boldsymbol{r}-\Delta\boldsymbol{r}(\boldsymbol{v},\boldsymbol{r},t-t')\\ \boldsymbol{v}'=\boldsymbol{v}-\Delta\boldsymbol{v}(\boldsymbol{v},\boldsymbol{r},t-t')}}$$

- **General form of dielectric tensor:** $\overleftrightarrow{\epsilon}=\overleftrightarrow{I}+\frac{\mathrm{i}}{\omega\epsilon_0}\overleftrightarrow{\sigma}$

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega)-\frac{\omega^2}{c^2}\int_V\mathrm{d}\boldsymbol{r}'\,\overleftrightarrow{\epsilon}(\boldsymbol{r},\boldsymbol{r}';\omega)\cdot\boldsymbol{E}(\boldsymbol{r}',\omega)-\mathrm{i}\,\omega\mu_0\boldsymbol{J}_{\mathrm{ext}}(\boldsymbol{r},\omega)=\boldsymbol{0}$$

Variable Transformations

- **Transformation of Integral variables**
  - Transformation from the velocity space variables $(v_\perp,\theta_g)$ to the particle position x' and guiding center position x_0.
  - Jacobian: $J=\dfrac{\partial(v_\perp,\theta_g)}{\partial(x',x_0)}=-\dfrac{\omega_c^2}{v_\perp\sin\omega_c\tau}$.
  - Express $v_\perp$ and θ_g by x' and x_0, e.g.,



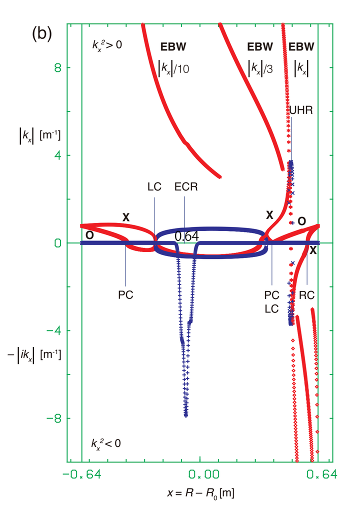
$$v_\perp\sin(\omega_c\tau+\theta_g)=\frac{\omega_c}{v_\perp}\frac{x-x'}{2}\frac{1}{\tan\frac{1}{2}\omega_c\tau}+\frac{\omega_c}{v_\perp}\left(\frac{x+x'}{2}-x_0\right)\tan\frac{1}{2}\omega_c\tau$$

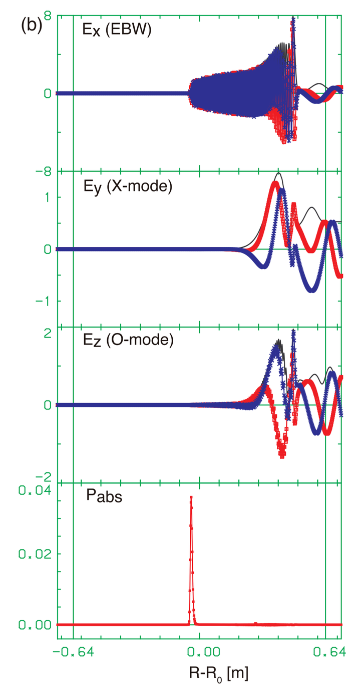
- **Integration over τ**: Fourier expansion with cyclotron motion
- **Integration over $v_\parallel$**: Plasma dispersion function
- **Conductivity tensor**: (for cyclotron harmonics)

$$\overleftrightarrow{\sigma}(x,x',\chi_0,\zeta_0)=-\mathrm{i}n_0\frac{q^2}{m}\sum_\ell\int\mathrm{d}x_0\,\overleftrightarrow{H}_\ell(x-x_0,x'-x_0;x_0,\chi_0,\zeta_0)$$

1D Kinetic FW Analysis of O-X-B Mode Conversion

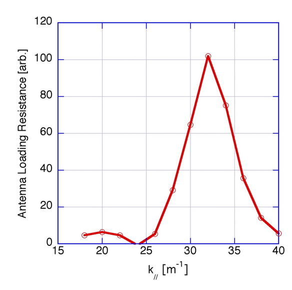
Dispersion relation | ***E* field & abs. profile** | **Parameters**





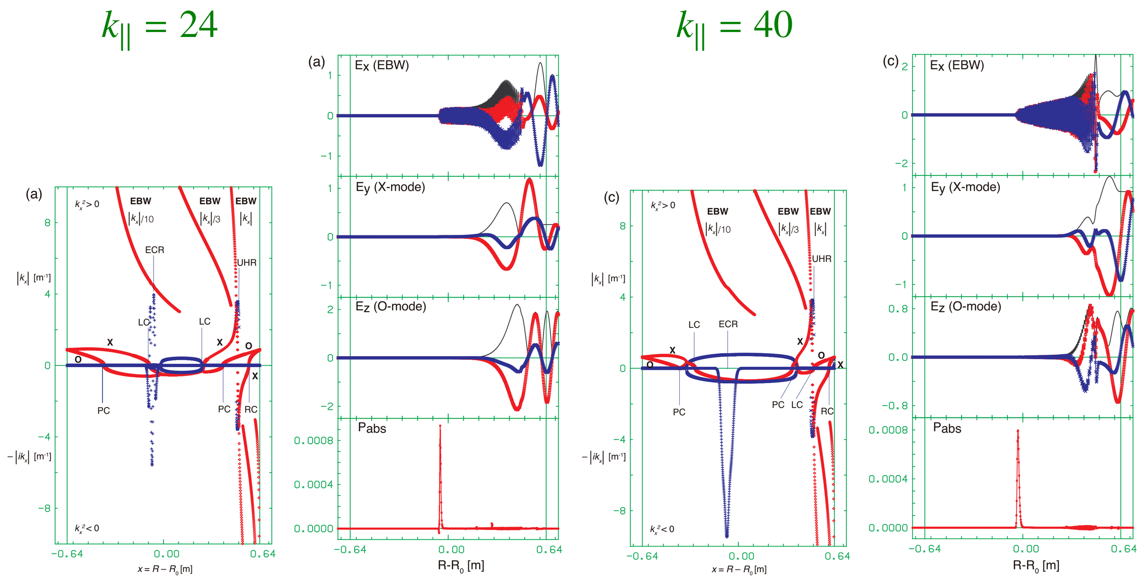
$R_0=0.22\,\mathrm{m}$
$a=0.15\,\mathrm{m}$
$B_0=0.08\,\mathrm{T}$
$n_e(0)=1\times10^{17}\,\mathrm{m}^{-3}$
$f=2.45\,\mathrm{GHz}$
$k_\parallel=32$
$T_e(0)=500\,\mathrm{eV}$

Wave number dependence of antenna load resistance



Parallel Wave Number Dependence

$k_\parallel=24$ | $k_\parallel=40$



Formulation for Cyclotron Resonance

- **Propagation along the magnetic field lines (mirror motion)**
- **Non-uniform magnetic field;** $B_z(z)=B_0\left(1+\frac{z}{L}\right)$
- **Basic equation**

$$\frac{1}{\beta^2}\nabla\times\nabla\times\boldsymbol{E}(z)-\int_{-\infty}^{\infty}\mathrm{d}z'\,\overleftrightarrow{\epsilon}(z-z')\boldsymbol{E}(z')=0$$

- **Dielectric tensor**

$$\overleftrightarrow{\epsilon}(z,z')=\delta(z-z')\overleftrightarrow{I}+\mathrm{i}\frac{\omega_{p0}^2}{\omega^2}\begin{pmatrix}(\chi_++\chi_-/2) & -\mathrm{i}\,(\chi_+-\chi_-/2) & 0\\ \mathrm{i}\,(\chi_+-\chi_-/2) & (\chi_++\chi_-/2) & 0\\ 0 & 0 & \chi_0\end{pmatrix}$$

$$\chi_\pm=\frac{(1+\kappa z)^{3/2}(1+\kappa z')^{3/2}}{(1+\kappa(z+z')/2)^2}U_0(\xi_\pm);\quad \kappa=\frac{v_{th}}{\omega L}$$

$$\chi_0=\frac{(1+\kappa z)(1+\kappa z')}{(1+\kappa(z+z')/2)}\left[\xi U_{-2}(\xi)-\frac{\kappa^2}{2(1+\kappa(z+z')/2)^2}\right]$$

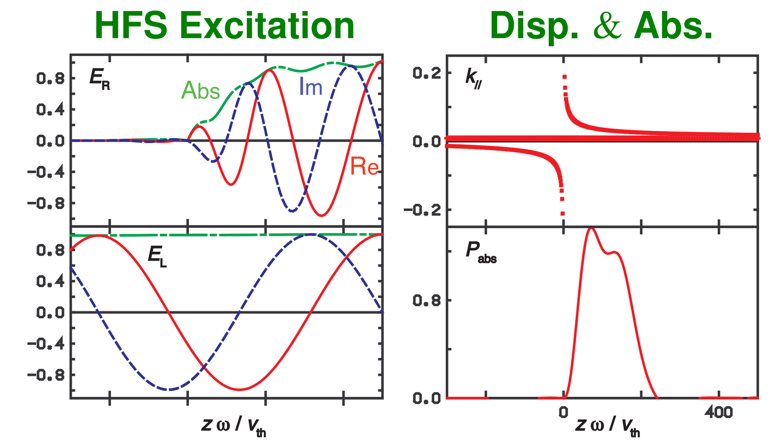
$$\xi=\frac{\omega(z-z')}{v_{th}},\quad \xi_\pm=\frac{(\omega\pm\Omega)(z-z')}{v_{th}},\quad \Omega=\frac{qB_0}{m}\left(1+\frac{z+z'}{2L}\right)$$

- **Kernel function**

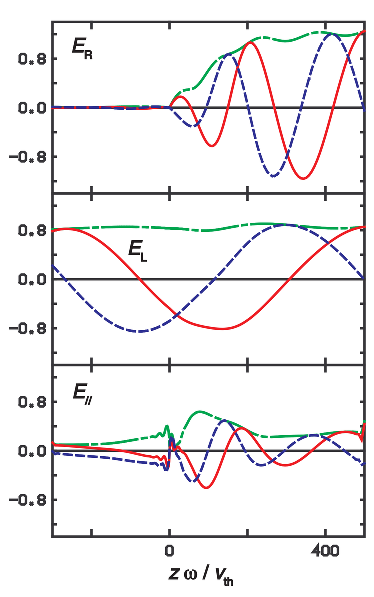
$$U_n(\xi)=\frac{1}{\sqrt{2\pi}}\int_0^\infty\tau^{n-1}\mathrm{d}\tau\exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2}+\mathrm{i}\,\tau\right]$$

Linearly Polarized Waves

HFS Excitation | **Disp. & Abs.**



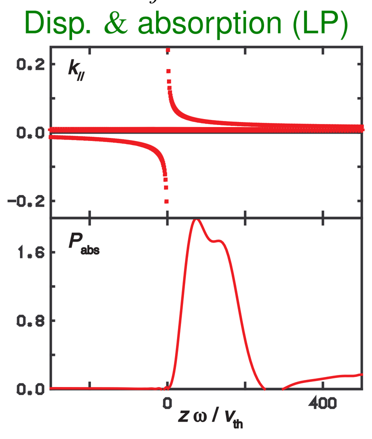
For non-zero $k_\perp$



Parameters

$\omega_{ce}=\omega$ at $z=0$
$\mathrm{d}B/\mathrm{d}z=5\times10^{-4}$
$\omega_{pe}^2/\omega^2=0.5$
$n_y=0.1$

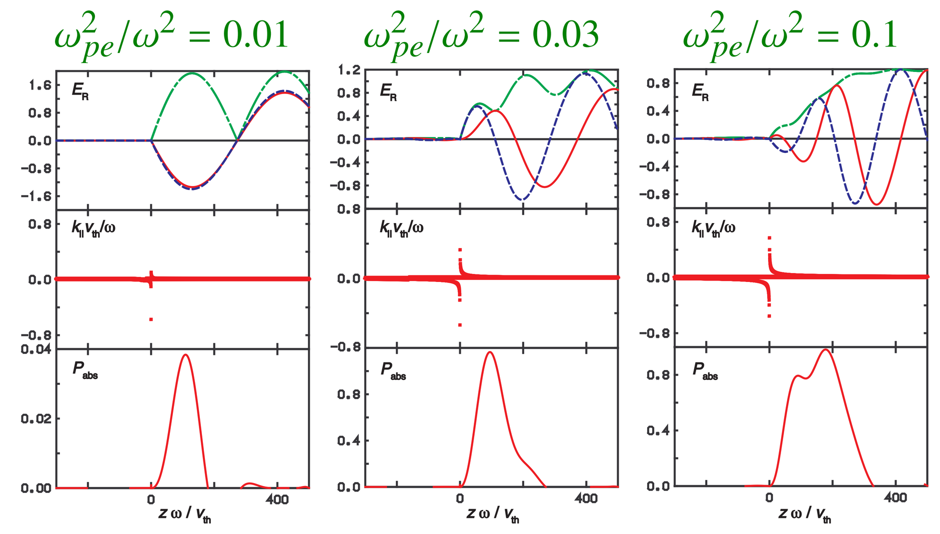
Disp. & absorption (LP)

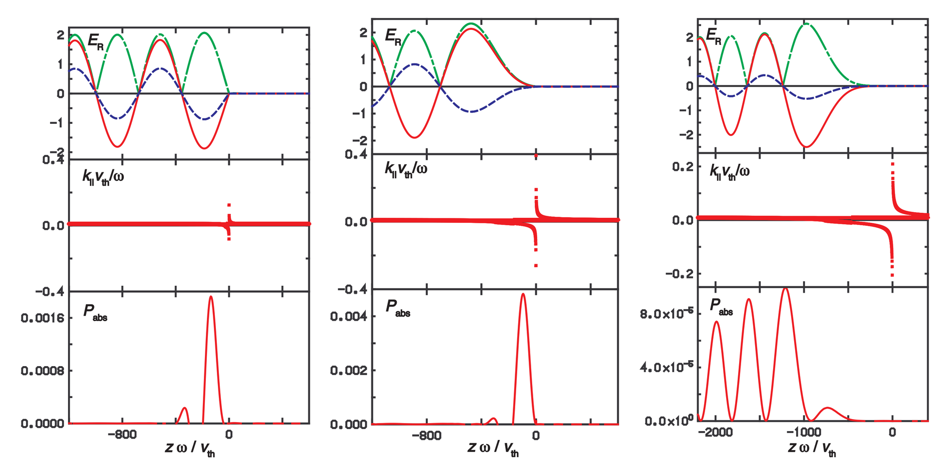


Circularly Polarized Waves (HFS & LFS)

- **Dependence on density:** $\beta=0.01$

$\omega_{pe}^2/\omega^2=0.01$ | $\omega_{pe}^2/\omega^2=0.03$ | $\omega_{pe}^2/\omega^2=0.1$





- **HFS:** Good absorption for high density
- **LFS:** Very weak absorption, no tunneling

Summary

- We have presented the **kinetic full wave analysis** of EC waves for **O-X-B mode conversion** following the particle orbit theory for 1D hot plasma.
- The analysis uses the **integral form of dielectric tensor** and describes the mode conversion to the **electron Bernstein wave** near the upper hybrid resonance (UHR) layer and absorption near the EC resonance.
- For parallel motion, **magnetic beach heating** near the EC resonance was studied and the power absorption profile was obtained.
- **2D kinetic full wave analysis** of O-X-B mode conversion is in progress. Preliminary results of the analysis are shown.