

3rd WORKSHOP ON PARTICLE PHYSICS

NATIONAL CENTRE FOR PHYSICS
(QUAID-I-AZAM UNIVERSITY)

Detectors for High Energy Physics

Lecture IV - Calorimetry

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Calorimetry

Calorimetry:

Energy measurement by **total absorption**, combined with spatial reconstruction.

Calorimetry is a “**destructive**” method **Detector response $\propto E$**

Calorimetry works both for

- ⇒ **charged** (e^\pm and hadrons)
- ⇒ and **neutral particles** (n, γ)

Basic mechanism: formation of

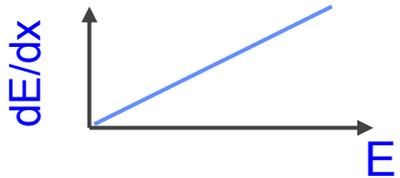
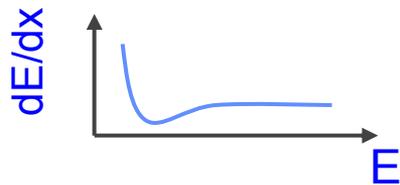
- ⇒ **electromagnetic**
- ⇒ or **hadronic showers.**

Finally, the energy is converted into ionization or excitation of the matter.

Basic electromagnetic interactions

e^+ / e^-

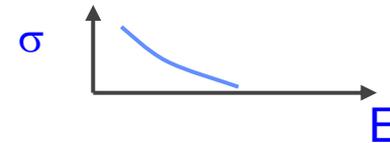
- Ionisation



- Bremsstrahlung

γ

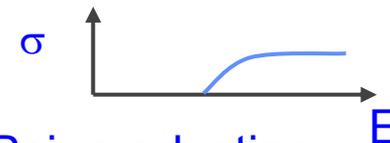
- Photoelectric effect



- Compton effect



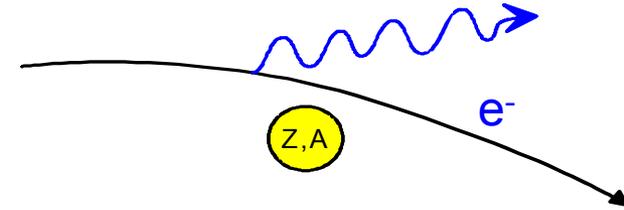
- Pair production



Energy loss by Bremsstrahlung

Radiation of real photons in the Coulomb field of the nuclei of the absorber

$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$



Effect plays a role only for e^\pm and ultra-relativistic μ (>1000 GeV)

For electrons:

$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}}$$

$$-\frac{dE}{dx} = \frac{E}{X_0}$$

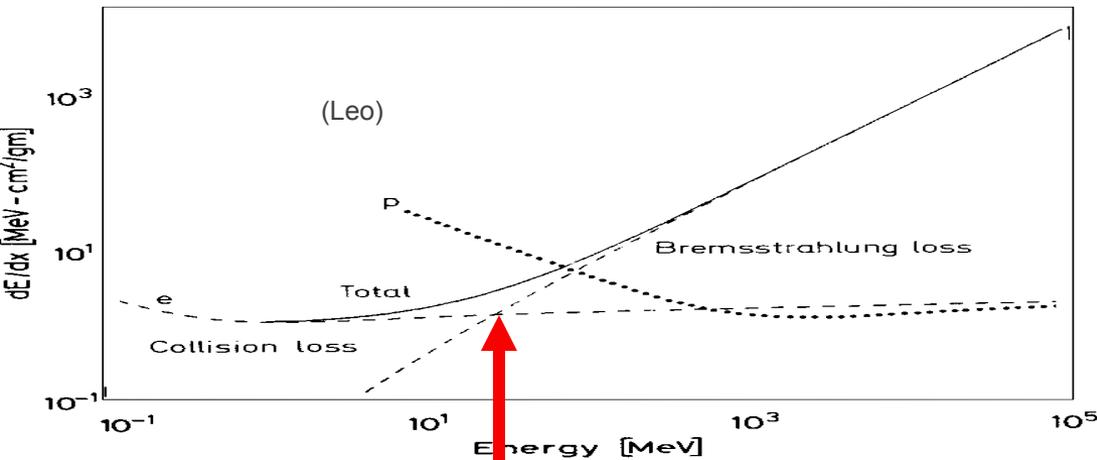


$$E = E_0 e^{-x/X_0}$$

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

← radiation length [g/cm²]

Critical energy



energy loss (radiative + ionization) of electrons and protons in copper

$$\left. \frac{dE}{dx} (E_c) \right|_{Brems} = \left. \frac{dE}{dx} (E_c) \right|_{ion}$$

for electrons

$$E_c^{solid + liq} = \frac{610 \text{ MeV}}{Z + 1.24}$$

$$E_c^{gas} = \frac{710 \text{ MeV}}{Z + 1.24}$$

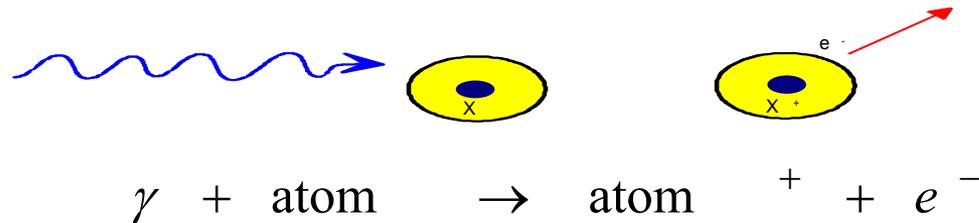
$$E_c \approx E_c^{elec} \left(\frac{m_\mu}{m_e} \right)^2$$

Ec(e-) in Fe(Z=26) = 22.4 MeV

Ec(m) in Fe(Z=26) ≈ 1 TeV

Photo-electric effect

In order to be detected, a photon has to create charged particles and/or transfer energy to charged particles

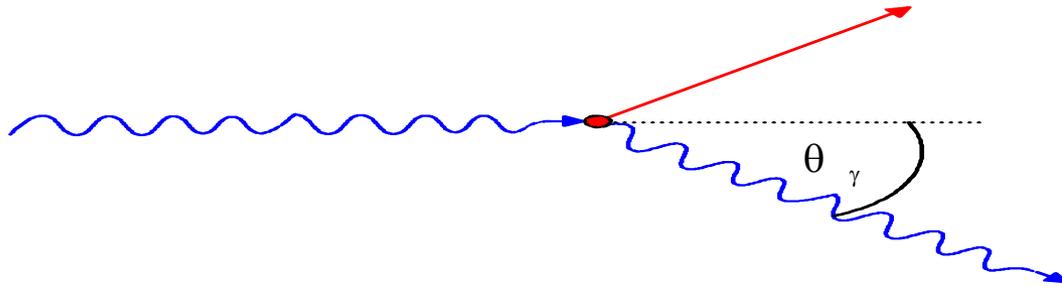


Only possible in the close neighborhood of a third collision partner \rightarrow photo effect releases mainly electrons from the K-shell

$$\sigma_{photo}^K = \left(\frac{32}{\epsilon^7} \right)^{\frac{1}{2}} \alpha^4 Z^5 \sigma_{Th}^e ; \quad \epsilon = \frac{E_\gamma}{m_e c^2} ; \quad \sigma_{Th}^e = \frac{8}{3} \pi r_e^2 \quad (\text{Thomson})$$

Compton scattering

$$\gamma + e \rightarrow \gamma' + e'$$



$$E'_\gamma = E_\gamma \frac{1}{1 + \varepsilon(1 - \cos\theta_\gamma)}$$

Assume electron as quasi-free.

Cross-section: Klein-Nishina formula, at high energies approximately

$$\sigma_c^e \propto \frac{\ln \varepsilon}{\varepsilon}$$

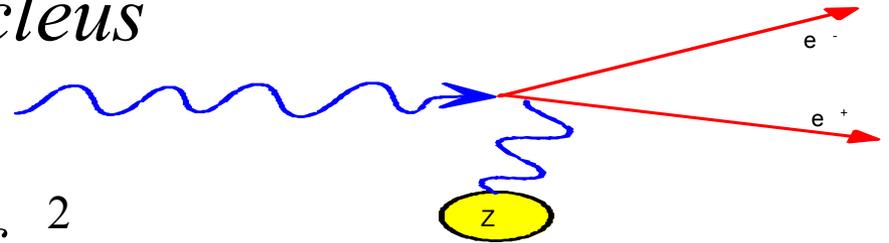
Atomic Compton cross-section:

$$\sigma_c^{atomic} = Z \cdot \sigma_c^e$$

Pair production

$$\gamma + nucleus \rightarrow e^+ e^- + nucleus$$

$$\text{only possible if } E_\gamma \geq 2 m_e c^2$$



$$\sigma_{pair} \approx 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{\frac{1}{3}}} \right)$$

independent of energy !

$$\approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

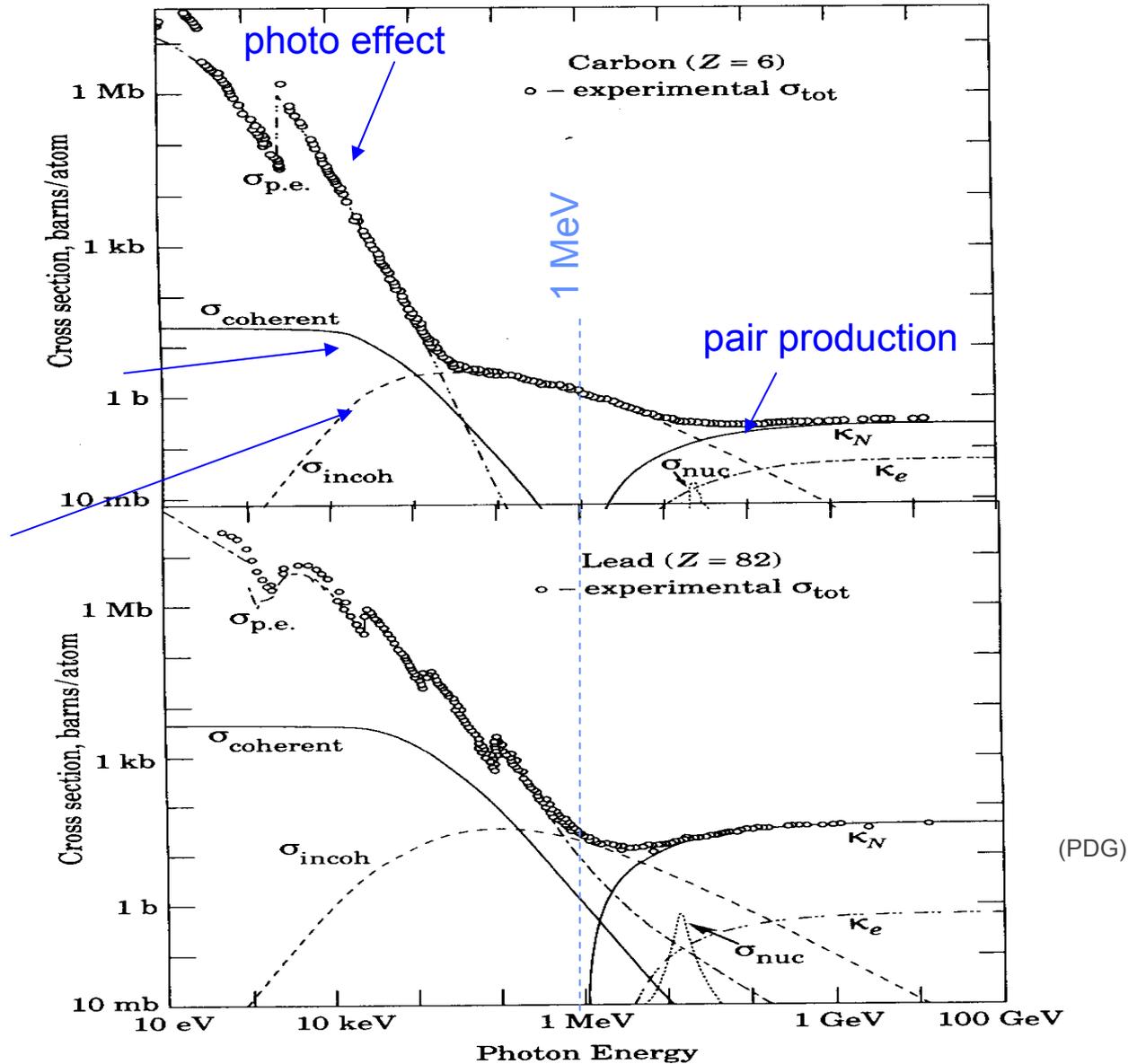
$$\approx \frac{A}{N_A} \frac{1}{\lambda_{pair}}$$

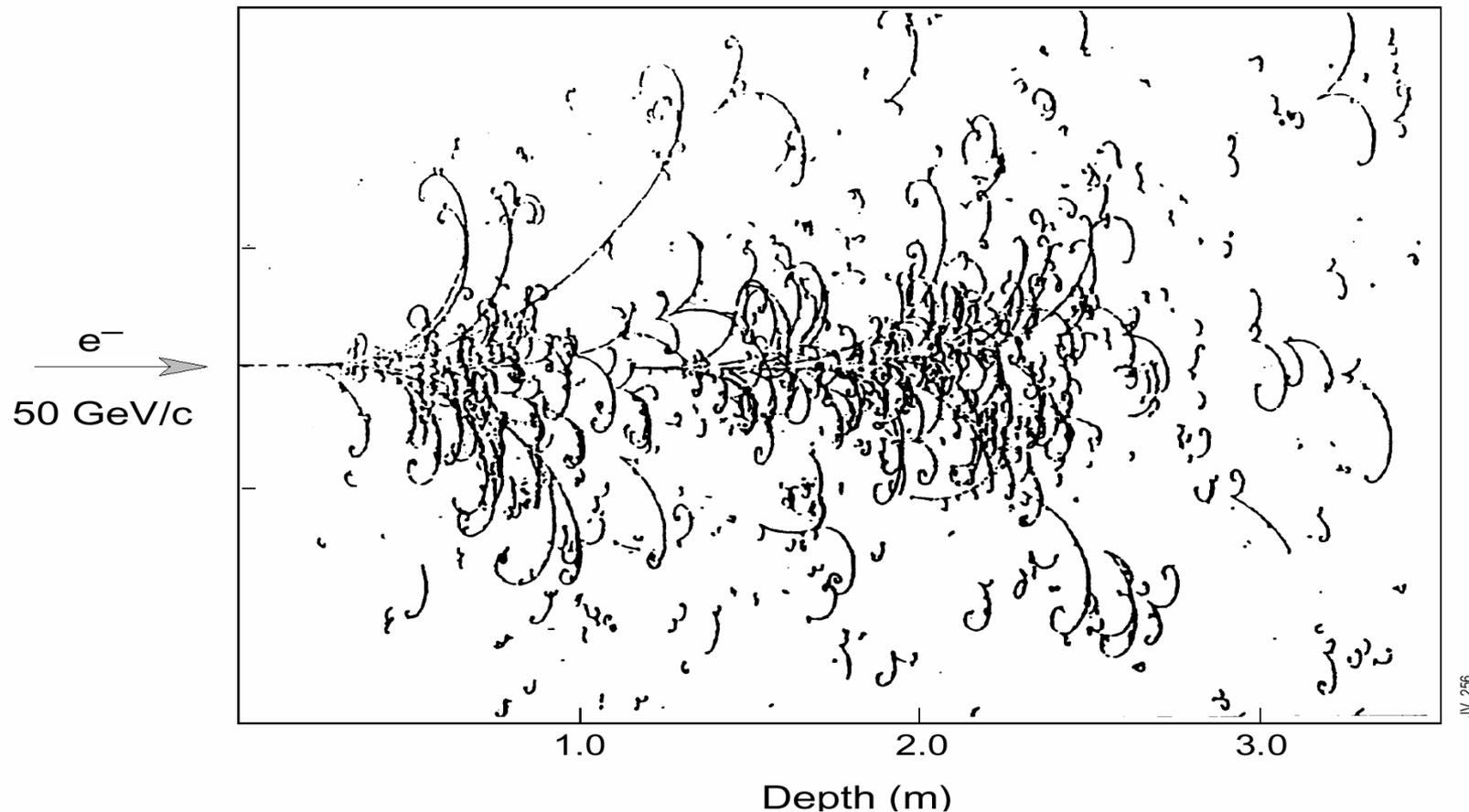
$$\sigma_{pair} = \frac{9}{7} X_0$$

Interaction of photons

Rayleigh scattering
(no energy loss !)

Compton scattering





**Big European Bubble Chamber filled with Ne:H₂ = 70%:30%,
3T Field, L=3.5 m, X₀≈34 cm, 50 GeV incident electron**

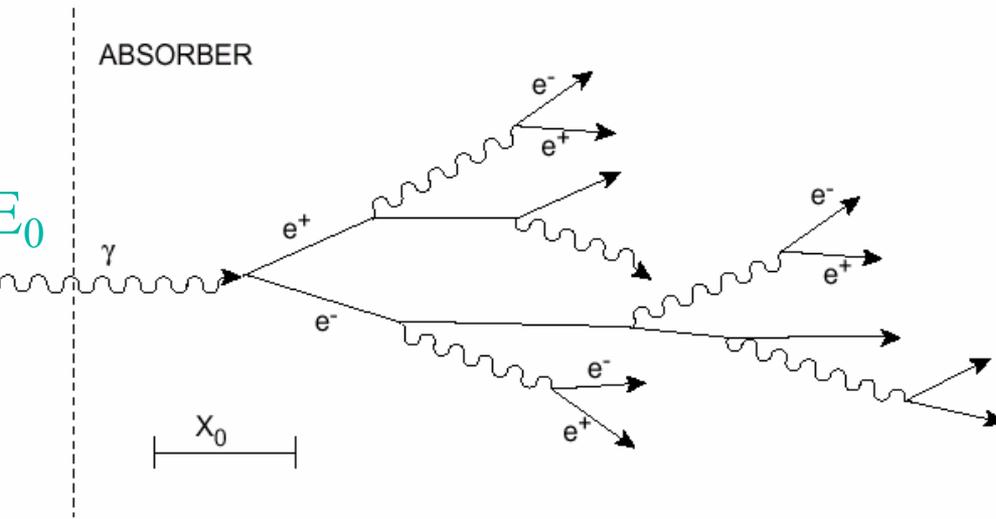
Above 1 GeV the dominant processes, bremsstrahlung for e^+ and e^- and pair production for γ , become energy independent

Through a succession of these energy loss mechanisms an electromagnetic cascade is propagated until the energy of charged secondaries has been degraded to the regime dominated by ionization loss (below E_c)

Below E_c a slow decrease in number of particles occurs as electrons are stopped and photons absorbed

Electromagnetic showers

A simple model



- In $1X_0$ an e loses about $2/3$ of its energy
- a high energy γ has a probability of $7/9$ of pair conversion
- Assume X_0 as a generation length
- In each generation the number of particles increases by a factor 2
- Until $E > E_c$

$\Delta x = X_0 \quad \gamma \rightarrow e^+ e^- \quad E = E_0/2 \quad @ \Delta x = 2X_0 \quad e \rightarrow \gamma e' \quad E' = E_0/4$

$\Delta x = tX_0 \quad N(t) = 2^t \quad E(t) = E_0 / 2^t$

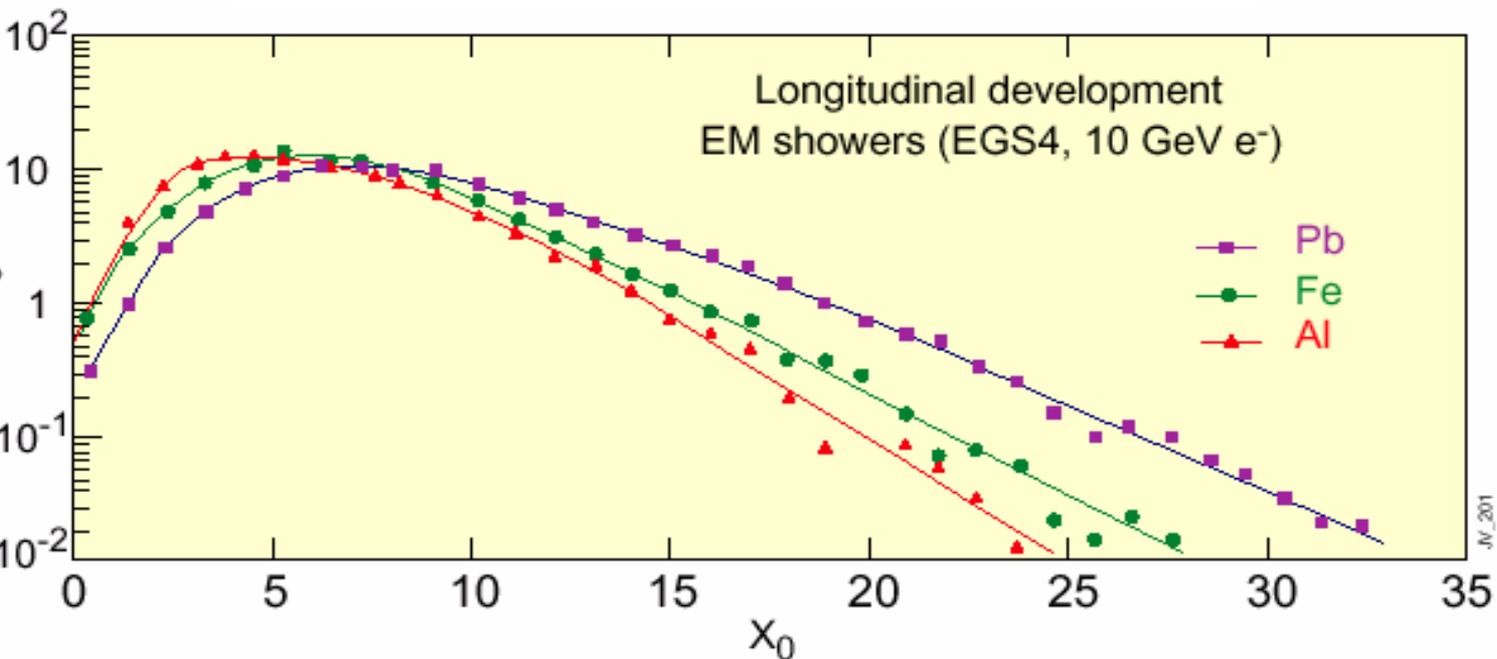
$t_{\max} X_0$ (shower max) $E(t_{\max}) = E_c \quad E_0 / 2^{t_{\max}} = E_c$

$$t_{\max} = \ln(E_0/E_c)/\ln(2)$$

$$N(t_{\max}) = 2 E_0/E_c - 1$$

Electromagnetic showers

Parametrization



$$\frac{dE}{dt} \propto t^\alpha e^{\beta t}$$

Parametrization of energy deposition

$$N_{\text{tot}} \propto E_0/E_c \quad t_{\text{max}} = 1.4 \ln(E_0/E_c)$$

Longitudinal containment

$$t_{95\%} = t_{\text{max}} + 0.08Z + 9.6$$

$$E_c \propto 1/Z$$



- shower max
- shower tail

Transverse shower profile

- Multiple scattering make electrons move away from shower axis
- Photons with energies in the region of minimal absorption can travel far away from shower axis

Molière radius sets transverse shower size, it gives the average lateral deflection of critical energy electrons after traversing $1X_0$

$$R_M = \frac{21\text{MeV}}{E_C} X_0$$

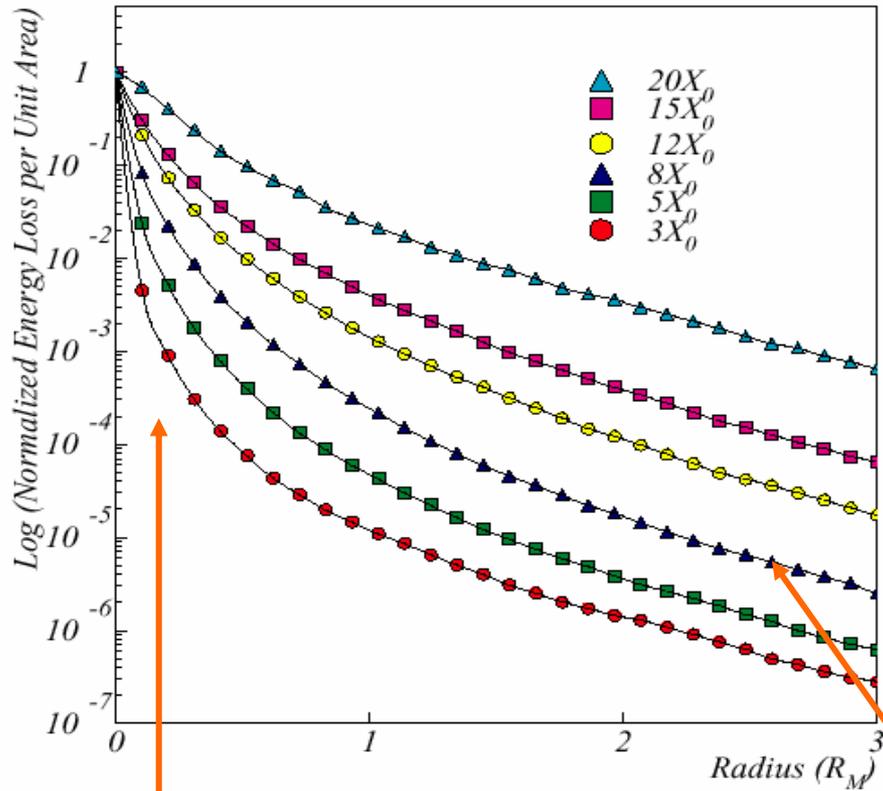
$$R_M \propto \frac{X_0}{E_C} \propto \frac{A}{Z} (Z \gg 1)$$

75% E_0 within $1R_M$, 95% within $2R_M$, 99% within $3.5R_M$

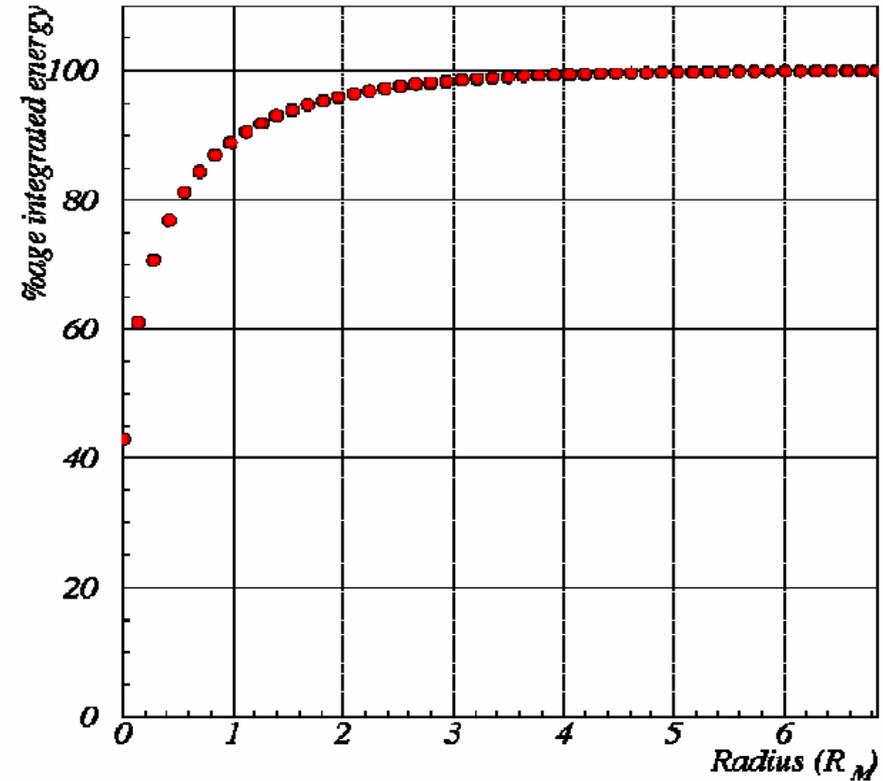
Electromagnetic showers

Parametrization

50 GeV electrons in $PbWO_4$



50 GeV electrons in $PbWO_4$



Central core: multiple scattering

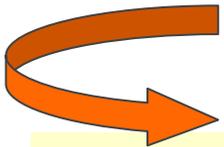
Peripheral halo: propagation of less attenuated photons, widens with depth of the shower

The energy deposited in the calorimeters is converted to active detector response

$$\bullet E_{\text{vis}} \leq E_{\text{dep}} \leq E_0$$

Main conversion mechanism

- Cerenkov radiation from e
- Scintillation from molecules
- Ionization of the detection medium



Different energy threshold E_s for signal detectability

Intrinsic limit

Detectable signal is proportional to the total track length of e^+ and e^- in the active material, intrinsic limit on energy resolution is given by the fluctuations in fraction of initial energy that generates detectable signal

$$N_{\text{tot}} \propto \frac{E_0}{E_C}$$

Total track length

$$T_0 = N_{\text{tot}} X_0 \approx \frac{E_0}{E_C} X_0$$

Detectable track length $T_r = f_s T_0$

f_s fraction of N_{tot} with $E > E_s$

Fluctuations in track length:

Poisson process

$$\frac{\sigma(E)}{E} \propto \frac{\sigma(T_r)}{T_r} \propto \frac{1}{\sqrt{T_r}} \propto \frac{1}{\sqrt{E_0}}$$

Calorimeter types

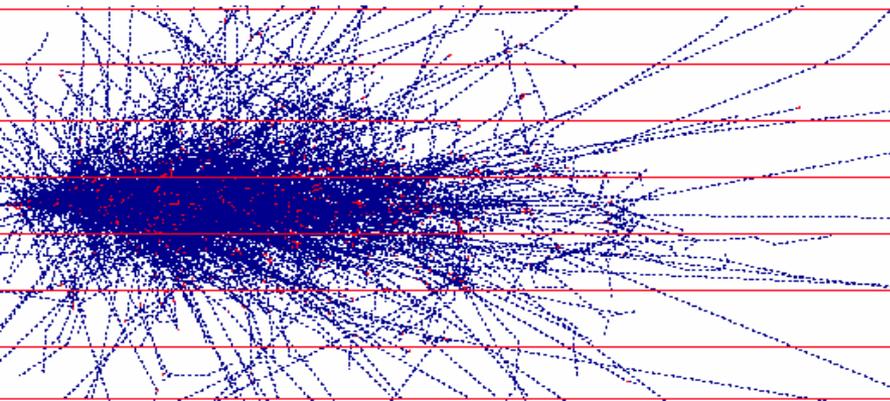
Homogeneous calorimeters:

- ⇒ Detector = absorber
- ⇒ good energy resolution
- ⇒ limited spatial resolution (particularly in longitudinal direction)
- ⇒ only used for electromagnetic calorimetry

Sampling calorimeters:

- ⇒ Detectors and absorber separated → only part of the energy is sampled.
- ⇒ limited energy resolution
- ⇒ good spatial resolution
- ⇒ used both for electromagnetic and hadron calorimetry

Homogeneous calorimeters: all the energy is deposited in the active medium. Absorber \equiv active medium



- Excellent energy resolution
- No information on longitudinal shower shape
- Cost

All e^+e^- over threshold produce a signal

$$\frac{\sigma(E)}{E} \propto \frac{1}{\sqrt{E}}$$

Compare processes with different energy threshold

Scintillating crystals

$$E_s \cong \beta E_{\text{gap}} \sim \text{eV}$$

$$\approx 10^2 \div 10^4 \gamma / \text{MeV}$$

$$\sigma / E \sim (1 \div 3)\% / \sqrt{E(\text{GeV})}$$



Lowest possible limit

Cherenkov radiators

$$\beta > \frac{1}{n} \rightarrow E_s \sim 0.7 \text{MeV}$$

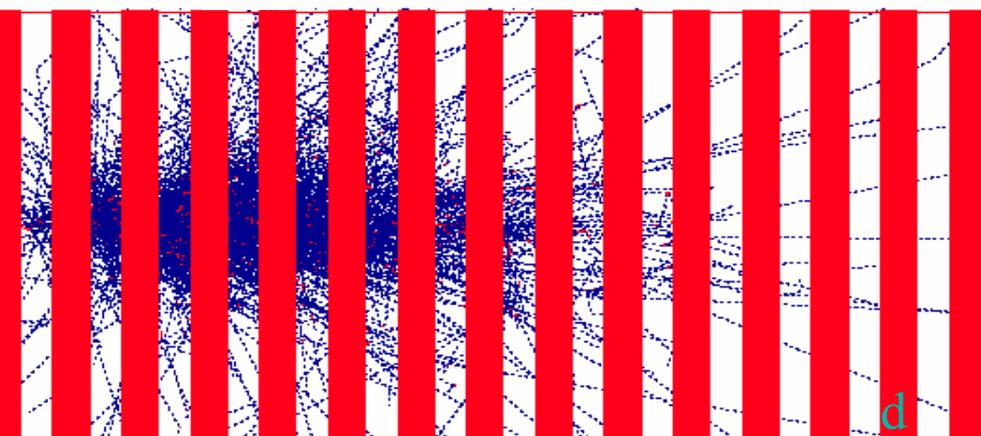
$$\approx 10 \div 30 \gamma / \text{MeV}$$

$$\sigma / E \sim (10 \div 5)\% / \sqrt{E(\text{GeV})}$$

Crystal Calorimeters: scintillators

Scintillator	Density [g/cm ³]	X ₀ [cm]	Light Yield γ/MeV (rel. yield)	τ ₁ [ns]	λ ₁ [nm]	Rad. Dam. [Gy]	Comments
NaI (Tl)	3.67	2.59	4×10 ⁴	230	415	≥10	hygroscopic, fragile
CsI (Tl)	4.51	1.86	5×10 ⁴ (0.49)	1005	565	≥10	Slightly hygroscopic
CSI pure	4.51	1.86	4×10 ⁴ (0.04)	10 36	310 310	10 ³	Slightly hygroscopic
BaF ₂	4.87	2.03	10 ⁴ (0.13)	0.6 620	220 310	10 ⁵	
BGO	7.13	1.13	8×10 ³	300	480	10	
PbWO ₄	8.28	0.89	≈100	10 10	≈440 ≈530	10 ⁴	light yield =f(T)

Sampling calorimeters: shower is sampled by layers of active medium (low-Z) alternated with dense radiator (high-Z) material.



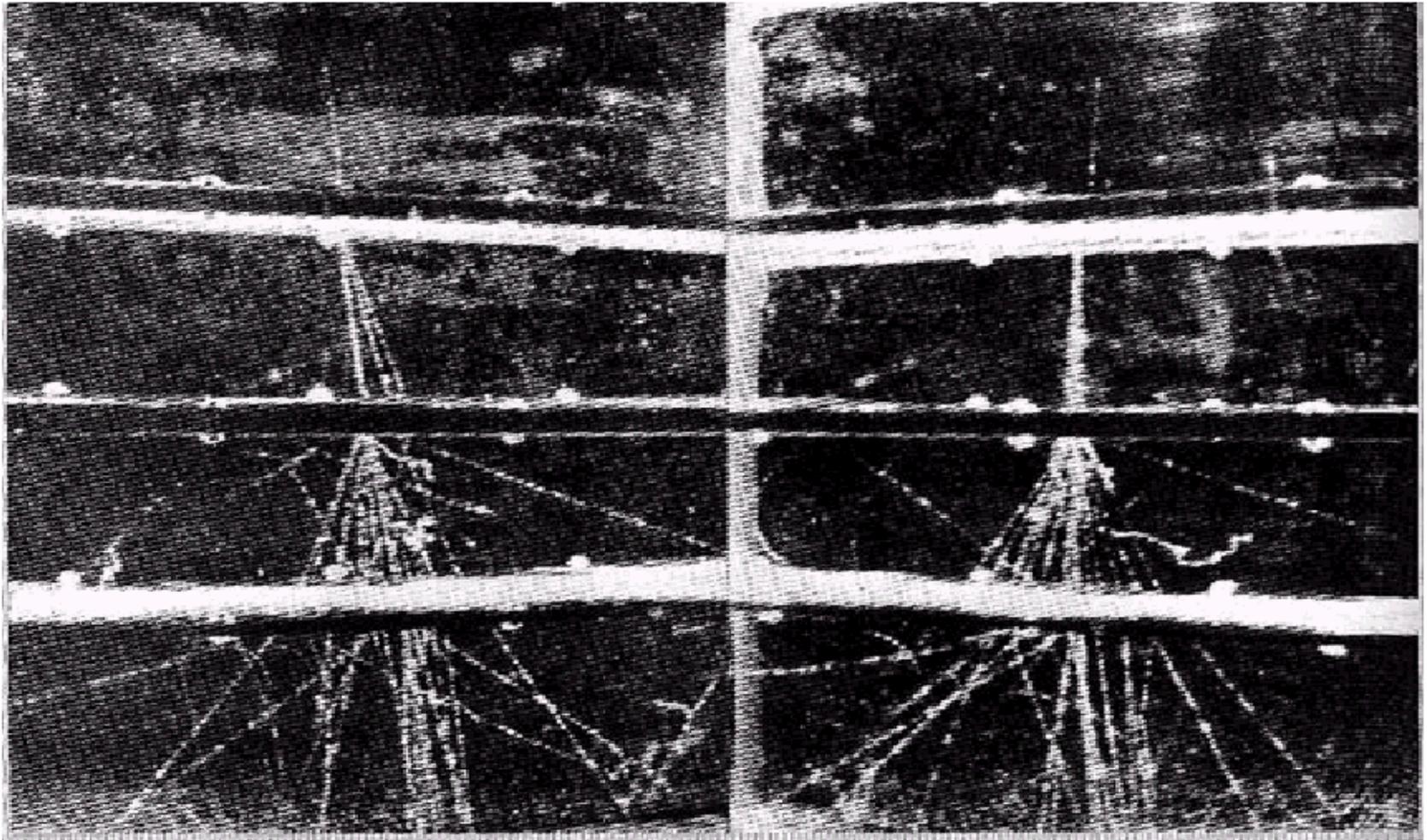
- Limited energy resolution
- Detailed shower shape information
- Cost

Shower generator separates active layers by a distance d



only a fraction of the shower energy is dissipated in the active medium
energy resolution is dominated by fluctuations in energy deposited in active layers: sampling fluctuations

Sampling electromagnetic showers

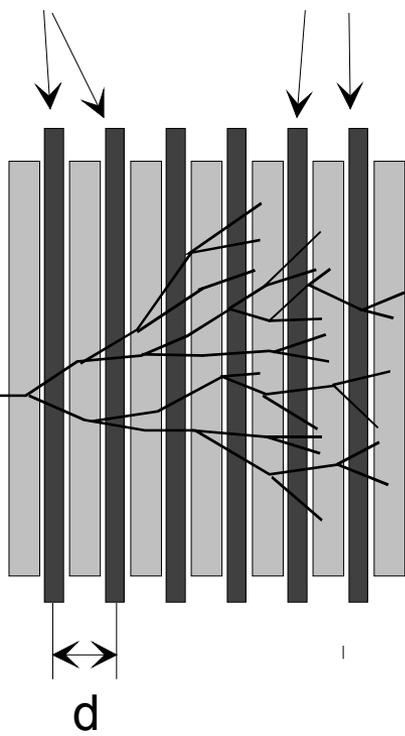


Cloud chamber photograph of e.m. shower developing in lead plates (thickness from top down 1.1, 1.1, 0.13 X_0) exposed to cosmic radiation

Electromagnetic calorimeters

Energy resolution

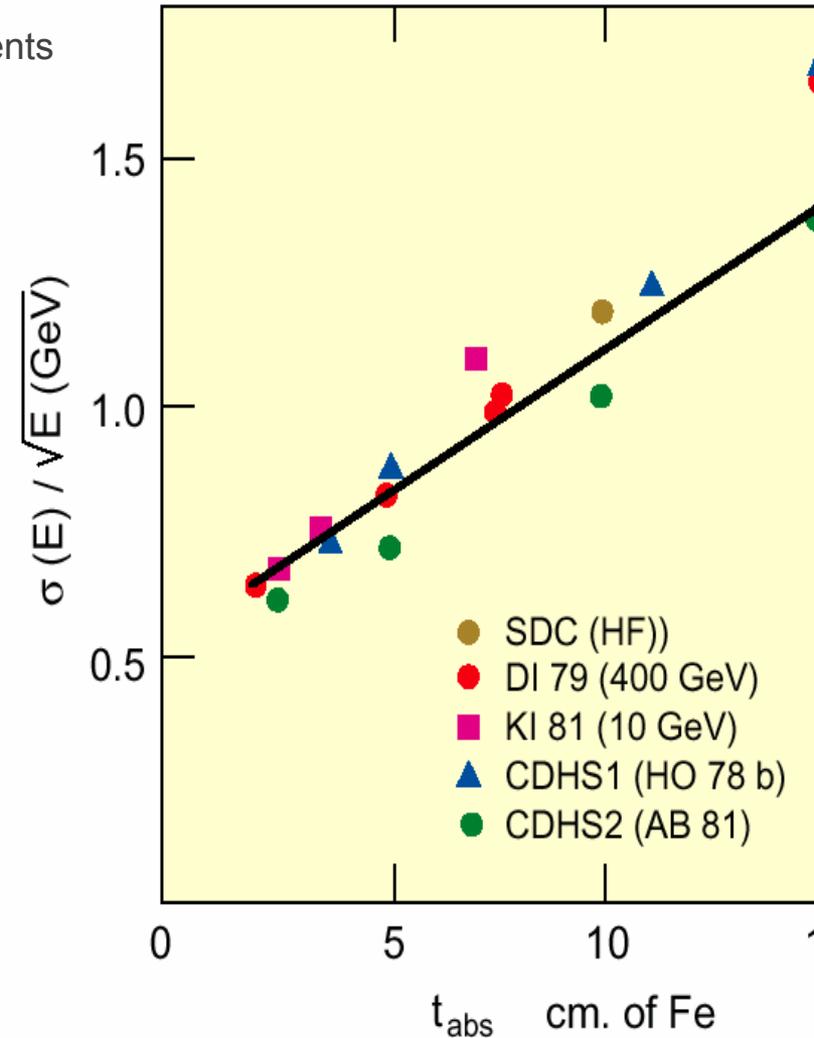
detectors absorbers



$$N = \frac{T_{\text{det}}}{d} \quad \text{Detectable track segments}$$

$$= F(\xi) \frac{E}{E_c} X_0 \frac{1}{d}$$

$$\frac{\sigma(E)}{E} \propto \frac{\sqrt{N}}{N} \propto \sqrt{\frac{1}{E}} \cdot \sqrt{\frac{d}{X_0}}$$

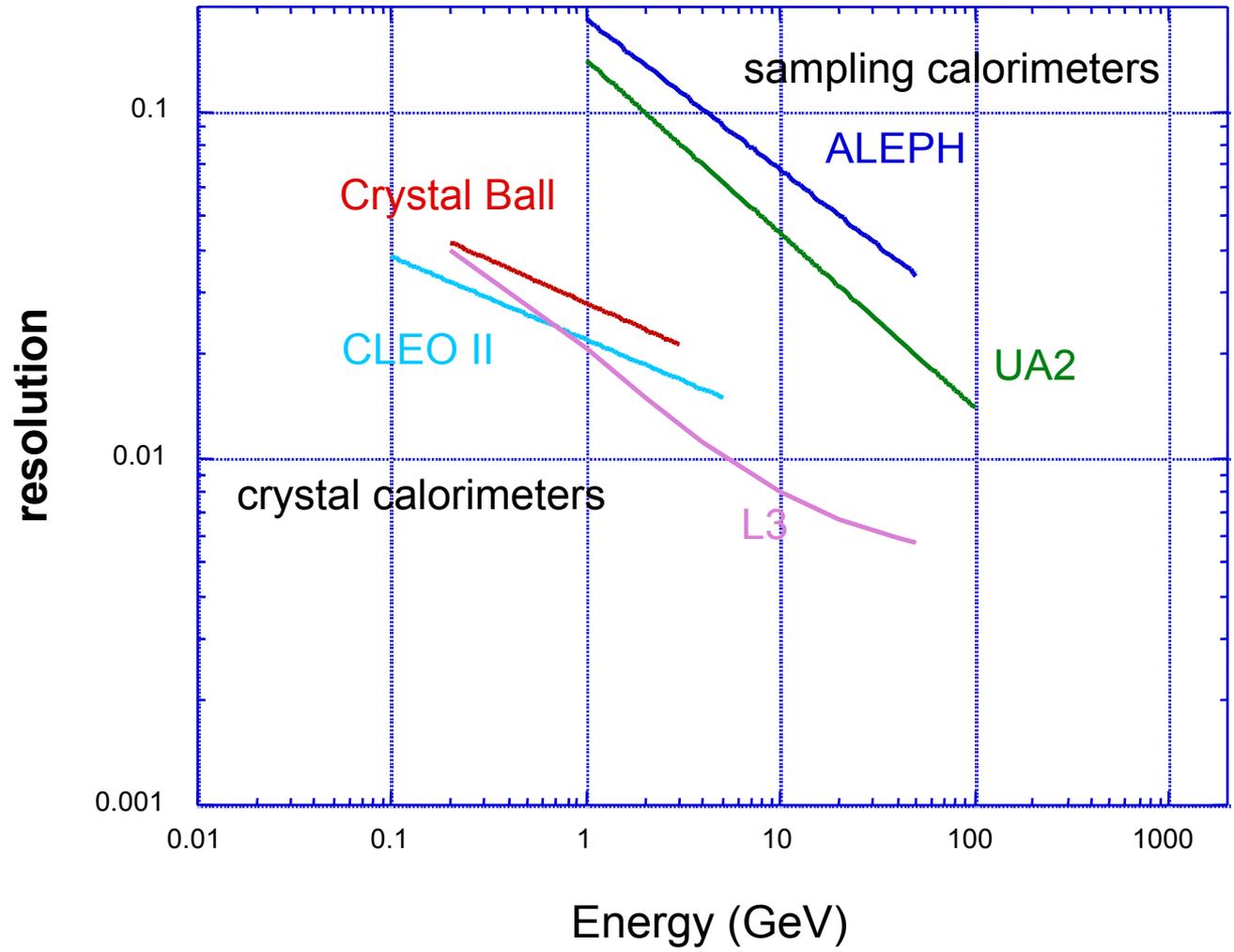


Energy resolution of a calorimeter can be parametrised as

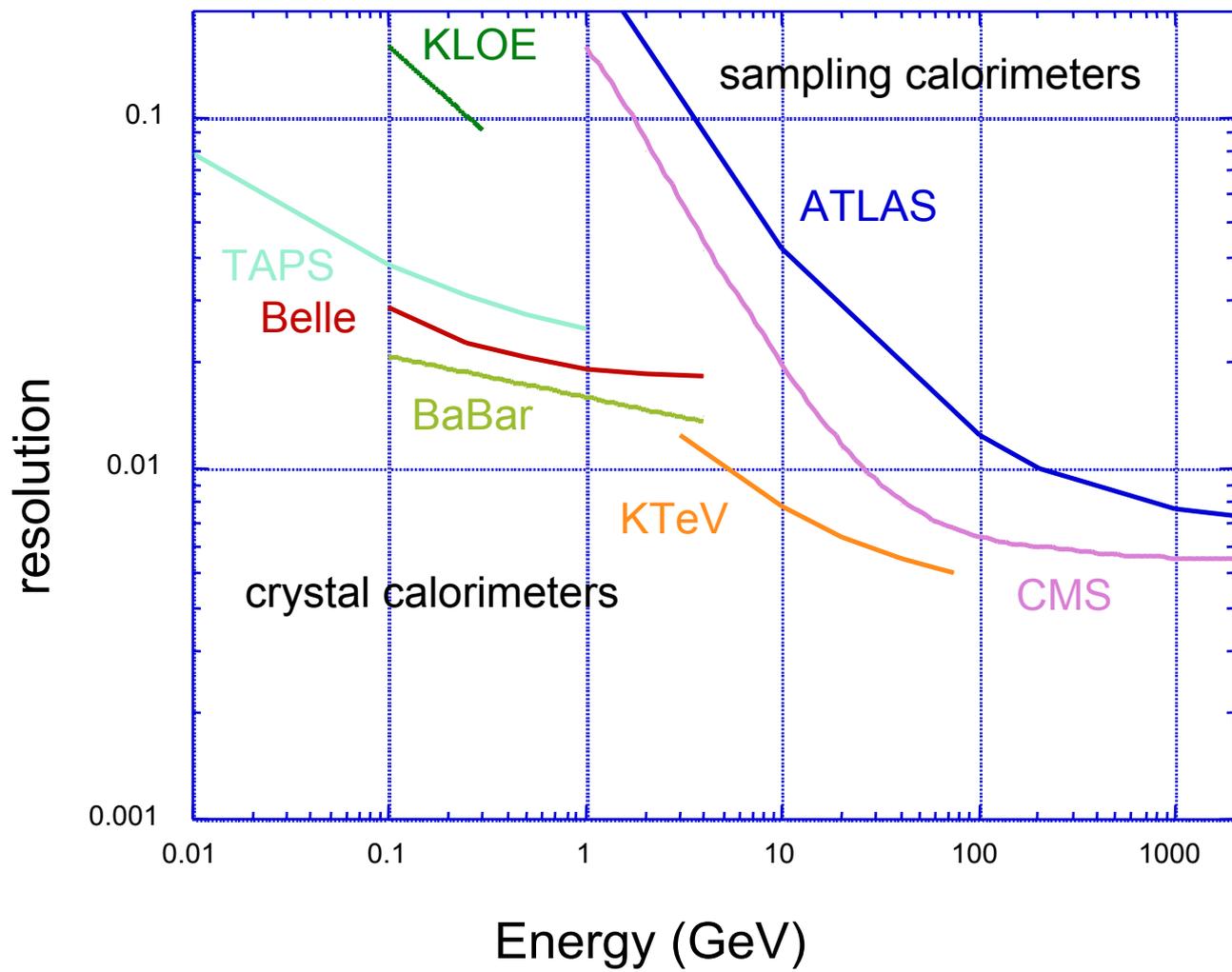
$$\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c \quad \oplus \text{ means quadratic sum}$$

- a the *stochastic term* accounts for any kind of Poisson-like fluctuations
 - natural merit of homogeneous calorimeters
 - several contributions add to the "intrinsic one"
- b the *noise term* responsible for degradation of low energy resolution
 - mainly the energy equivalent of the electronic noise
 - contribution from pileup: the fluctuation of energy entering the measurement area from sources other than the primary particle
- c the *constant term* dominates at high energy
 - its relevance is strictly connected to the small value of a
 - it is mostly dominated by the stability of calibration
 - contributions from energy leakage, non uniformity of signal generation and/or collection, loss of energy in dead materials,...

Classic calorimeters

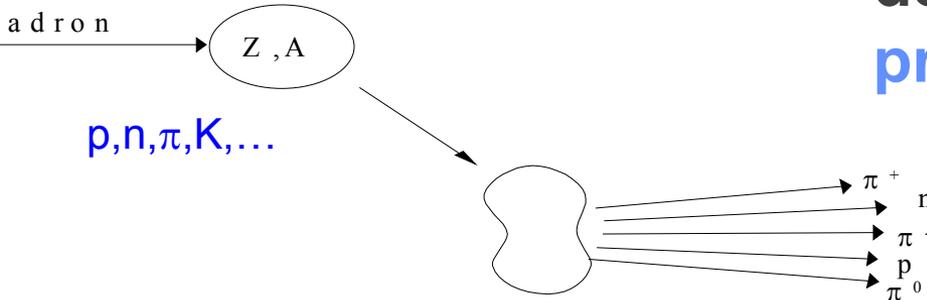


Modern calorimeters



Nuclear Interactions

The interaction of energetic hadrons (charged or neutral) is determined by **inelastic nuclear processes**.



multiplicity $\propto \ln(E)$

$$p_t \approx 0.35 \text{ GeV}/c$$

excitation and finally breakup up nucleus \rightarrow nucleus fragments + production of secondary particles.

For high energies ($>1 \text{ GeV}$) the cross-sections depend only little on the energy and on the type of the incident particle (p, pi, K...).

In analogy to X0 a hadronic absorption length can be defined

$$\lambda_a = \frac{A}{N_A \sigma_{inel}}$$

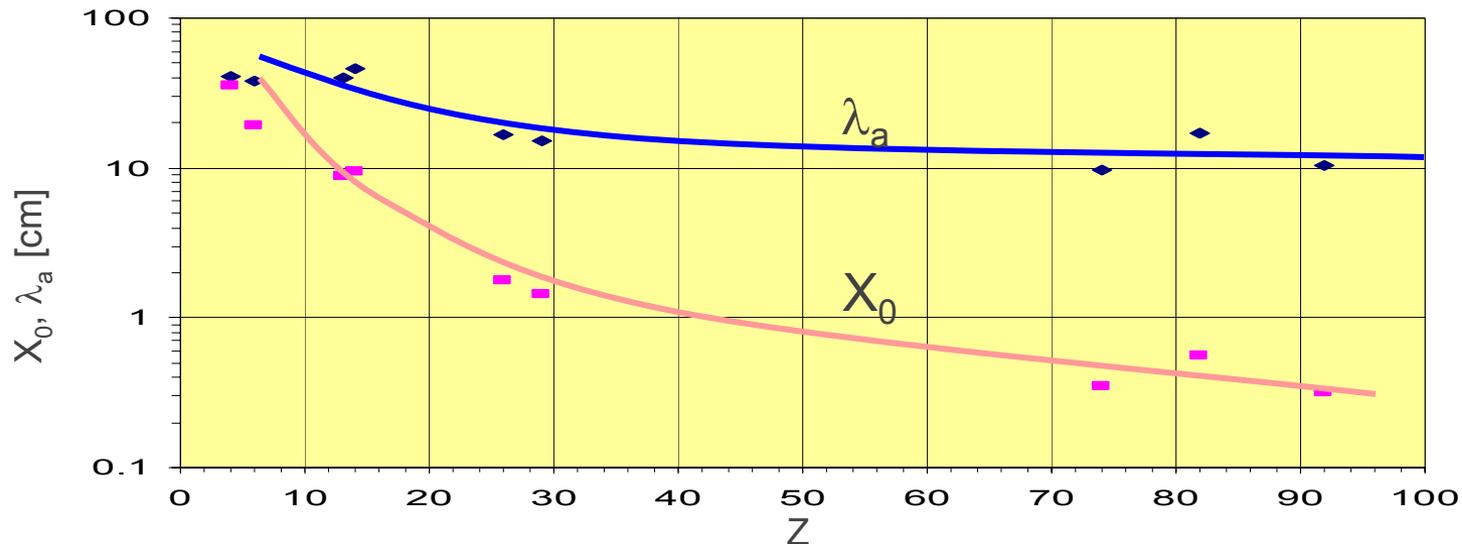
$$\sigma_{inel} \approx \sigma_0 A^{0.7} \quad \sigma_0 \approx 35 \text{ mb}$$

Interaction of charged particles

Material	Z	A	ρ [g/cm ³]	X_0 [g/cm ²]	λ_a [g/cm ²]
Hydrogen (gas)	1	1.01	0.0899 (g/l)	63	50.8
Helium (gas)	2	4.00	0.1786 (g/l)	94	65.1
Beryllium	4	9.01	1.848	65.19	75.2
Carbon	6	12.01	2.265	43	86.3
Nitrogen (gas)	7	14.01	1.25 (g/l)	38	87.8
Oxygen (gas)	8	16.00	1.428 (g/l)	34	91.0
Aluminium	13	26.98	2.7	24	106.4
Silicon	14	28.09	2.33	22	106.0
Iron	26	55.85	7.87	13.9	131.9
Copper	29	63.55	8.96	12.9	134.9
Tungsten	74	183.85	19.3	6.8	185.0
Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

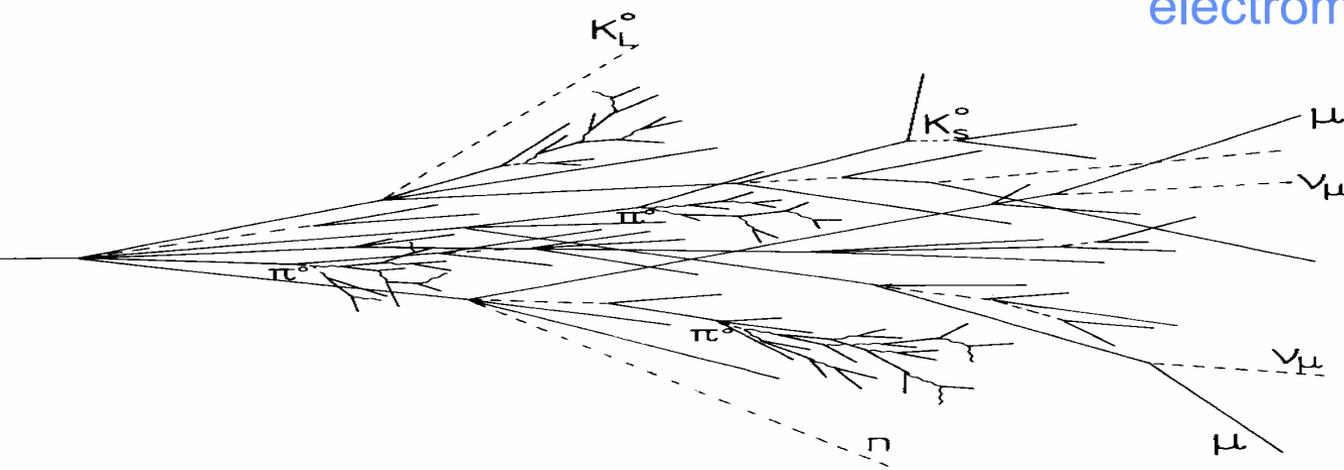
For $Z > 6$: $\lambda_a > X_0$

λ_a and X_0 in cm



Hadronic cascades

Various processes involved. Much more complex than electromagnetic cascades.



Hadronic

+

electromagnetic

component

(Grupe)

charged pions, protons, kaons

....

Breaking up of nuclei

(binding energy),

neutrons, neutrinos, soft γ 's

muons \rightarrow invisible energy

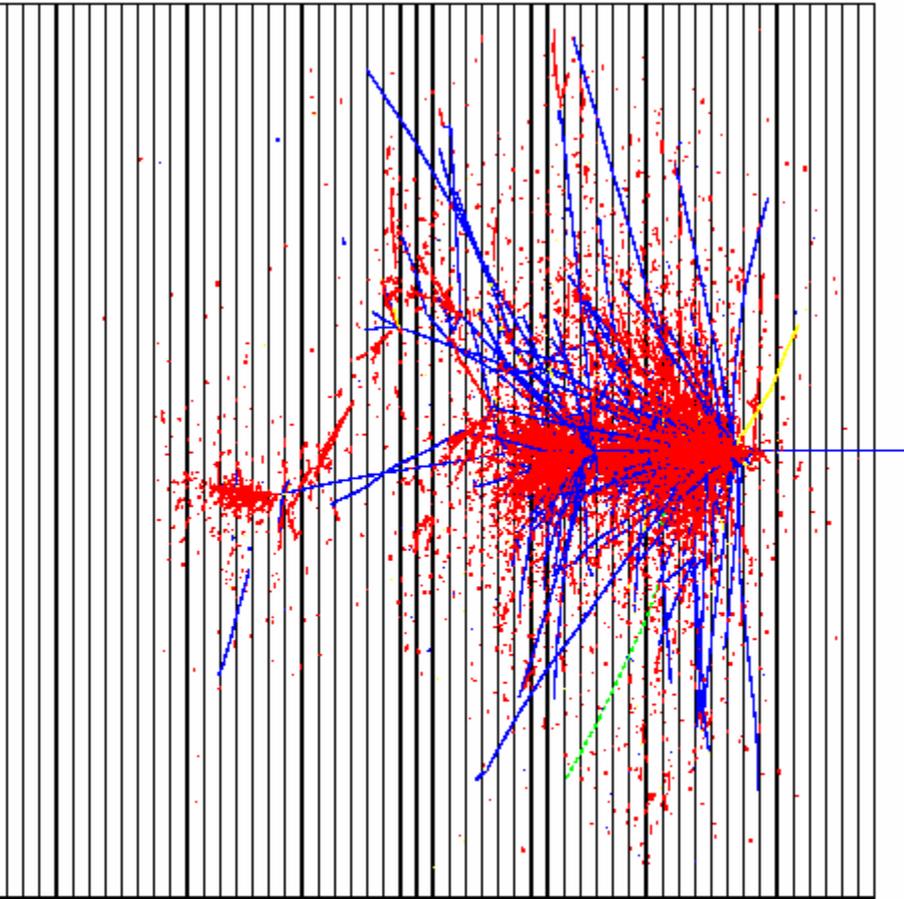
neutral pions $\rightarrow 2\gamma \rightarrow$ electromagnetic cascade

$$n(\pi^0) \approx \ln E(\text{GeV}) - 4.6$$

example 100 GeV: $n(\pi^0) \approx 18$

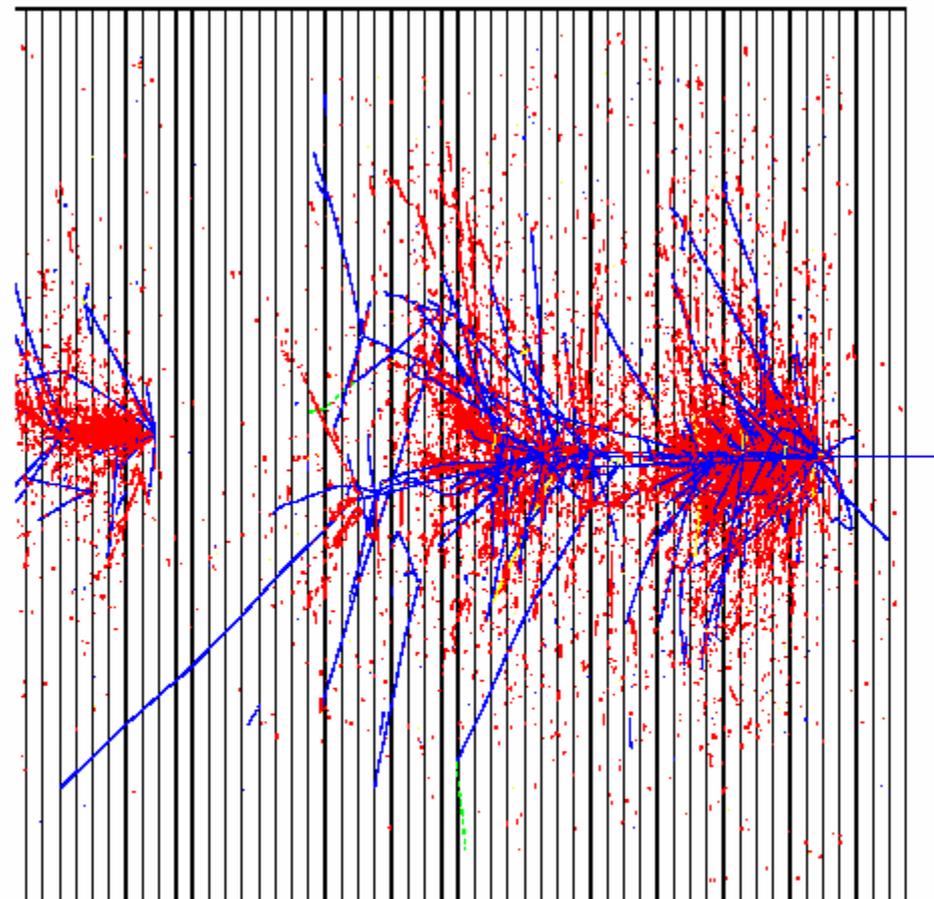
Large energy fluctuations \rightarrow limited energy resolution

150 GeV Pion Showers in Cu



Hadron shower not as well behaved as an em one

red - e.m. component
blue - charged hadrons



Hadron calorimeter are always sampling calorimeters

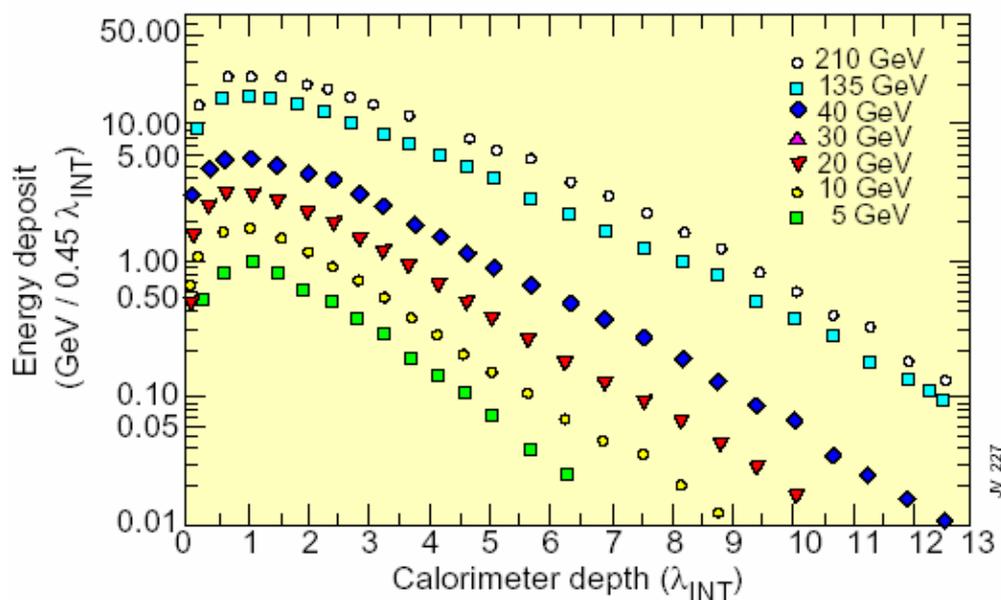
Hadronic Cascade: Profiles

Hadron shower profiles for single π^\pm

Longitudinal

- sharp peak from π^0 's produced in the 1st interaction
- followed by a more gradual falloff with a characteristic scale of λ .

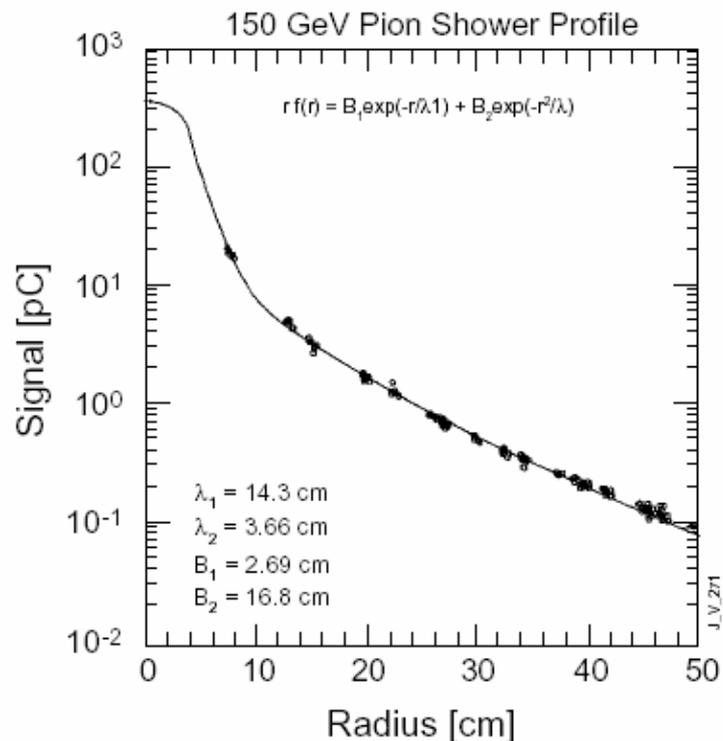
WA78 : 5.4λ of 10mm U / 5mm Scint + 8λ of 25mm Fe / 5mm Scint



Approx. 10λ required to contain 99% of the energy of ≈ 200 GeV pions

Lateral

- Secondaries produced with $\langle p_t \rangle \sim 300$ MeV -approx. energy lost in $\approx 1 \lambda$ in most materials.
- Characteristic transverse scale is $r_\pi \approx \lambda$.
- Pronounced core, caused by the π^0 component,

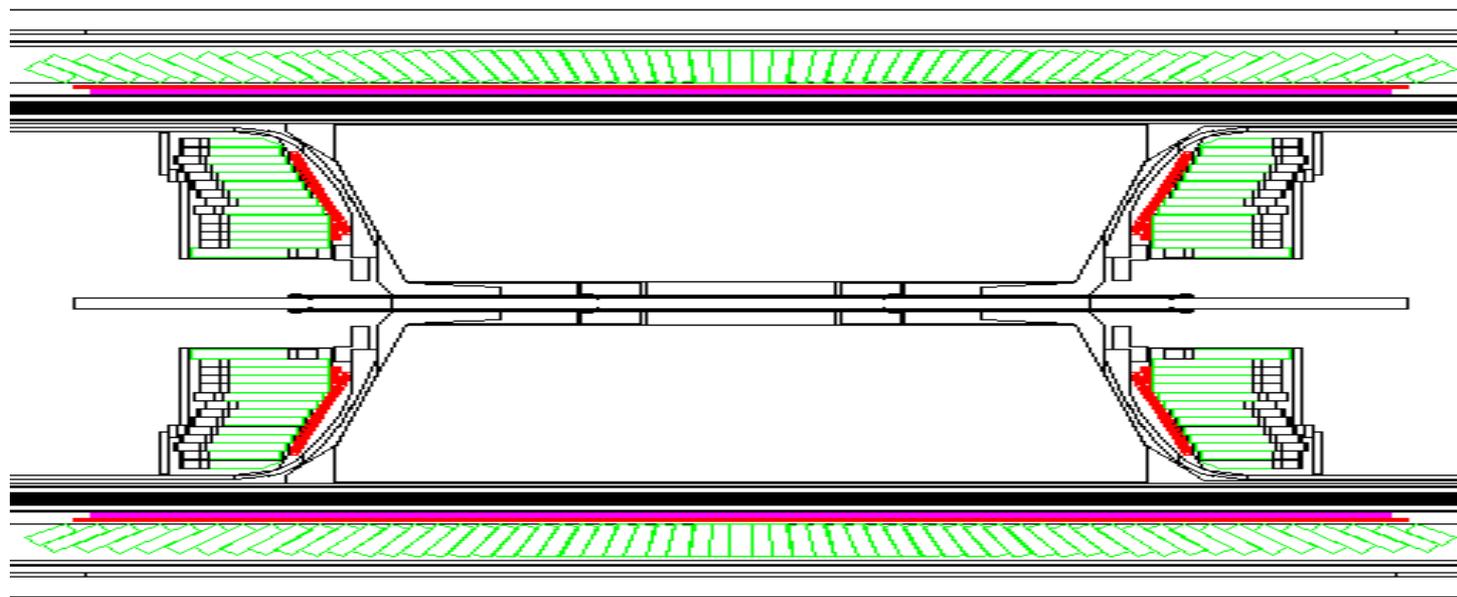


Transverse radius for 95% containment is $R_{0.95} \approx 1 \lambda$

Homogeneous calorimeters

OPAL Barrel + end-cap:
lead glass + pre-sampler

(OPAL collab. NIM A 305 (1991) 275)



≈ 10500 blocks ($10 \times 10 \times 37$ cm³, $24.6 X_0$),
PM (barrel) or PT (end-cap) readout.

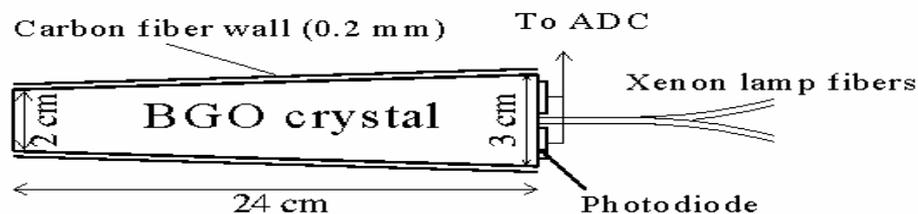
$$\sigma(E)/E = 0.06/\sqrt{E} \oplus 0.002$$

Spatial resolution
(intrinsic) ≈ 11
mm at 6 GeV

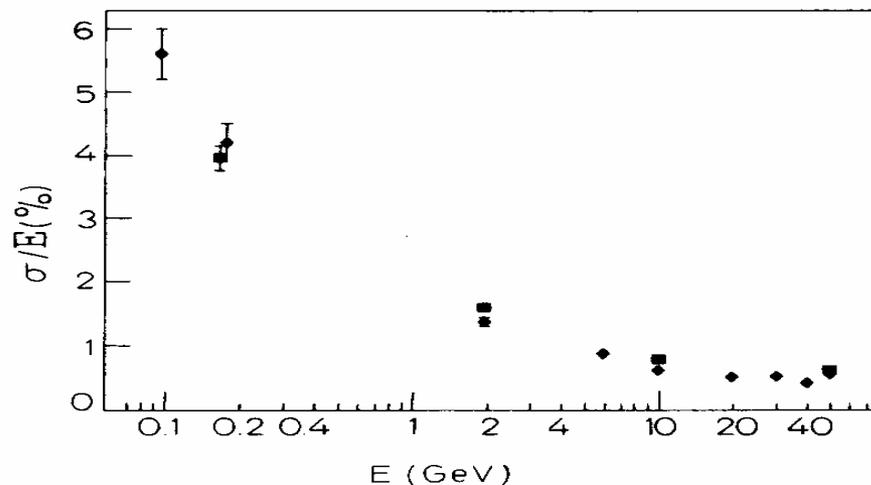
Homogeneous calorimeters

BGO E.M. Calorimeter in L3

(L3 collab. NIM A 289 (1991) 53)



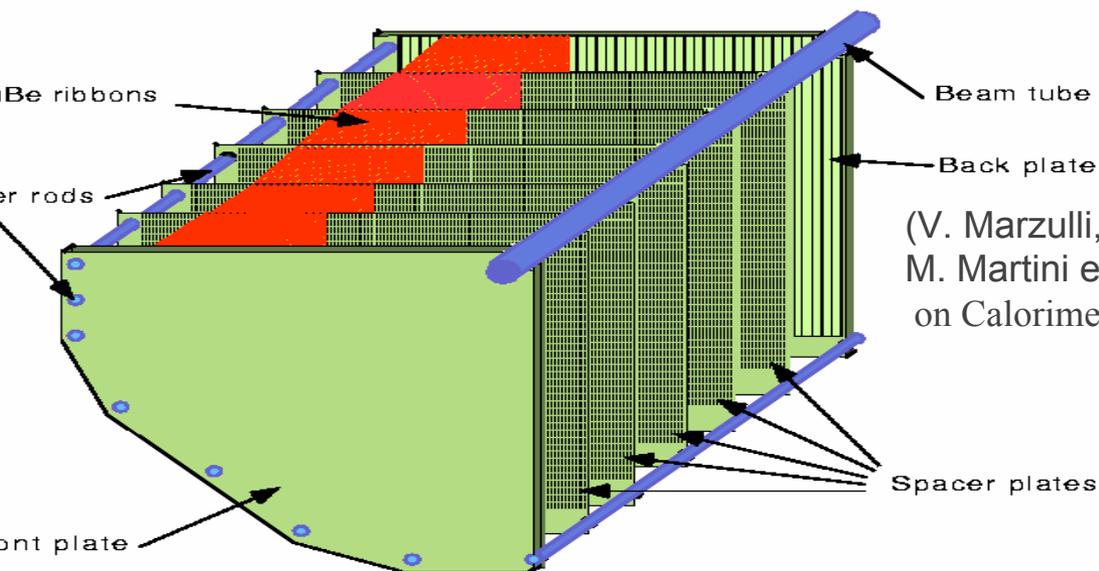
1000 crystals, 21.4 X_0 , temperature monitoring + control system
light output -1.55% / °C



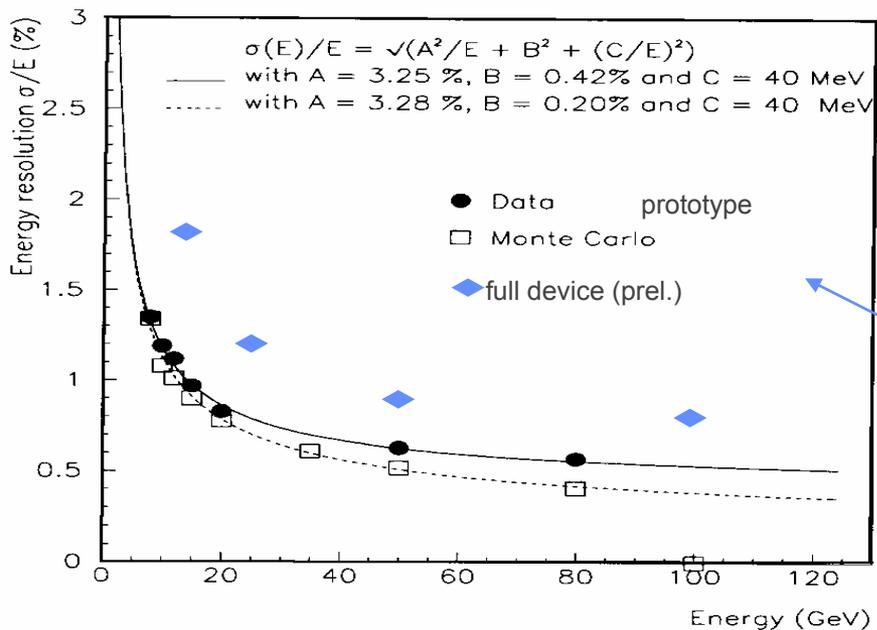
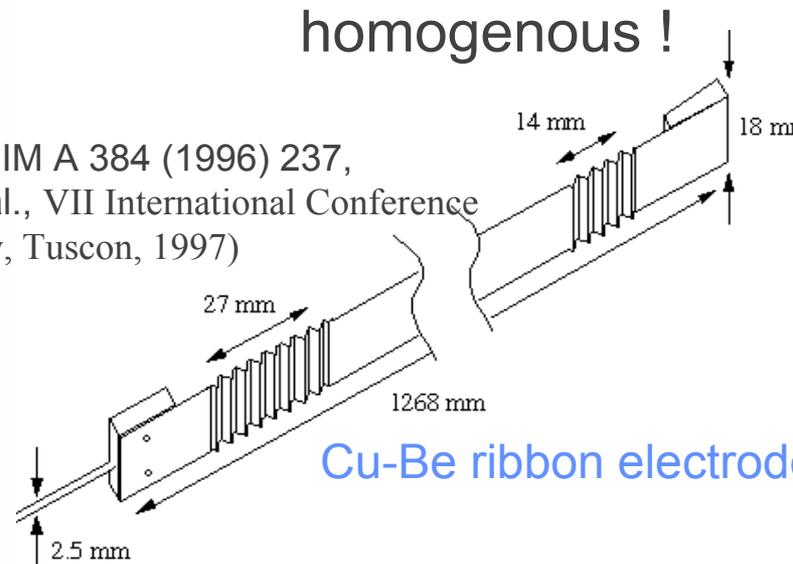
$\sigma_E/E < 1\%$ for $E > 1$ GeV
spatial resolution < 2 mm
($E > 2$ GeV)

Partly test beam results !

Homogeneous calorimeters



(V. Marzulli, NIM A 384 (1996) 237,
M. Martini et al., VII International Conference
on Calorimetry, Tuscon, 1997)



$\sigma_{x,y} \leq 1$ mm
 $\sigma_t \approx 230$ ps

97 run: reduced performance due to problem
with blocking capacitors → lower driftfield: 1
kV/cm rather than 5 kV/cm

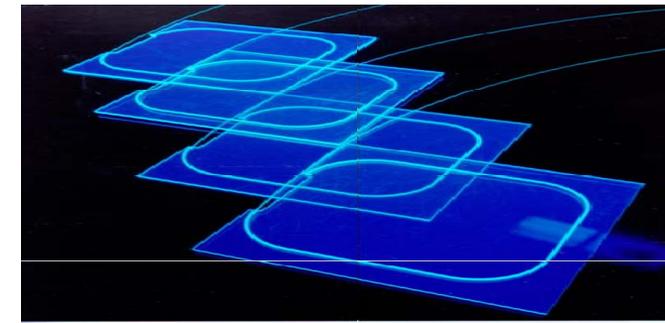
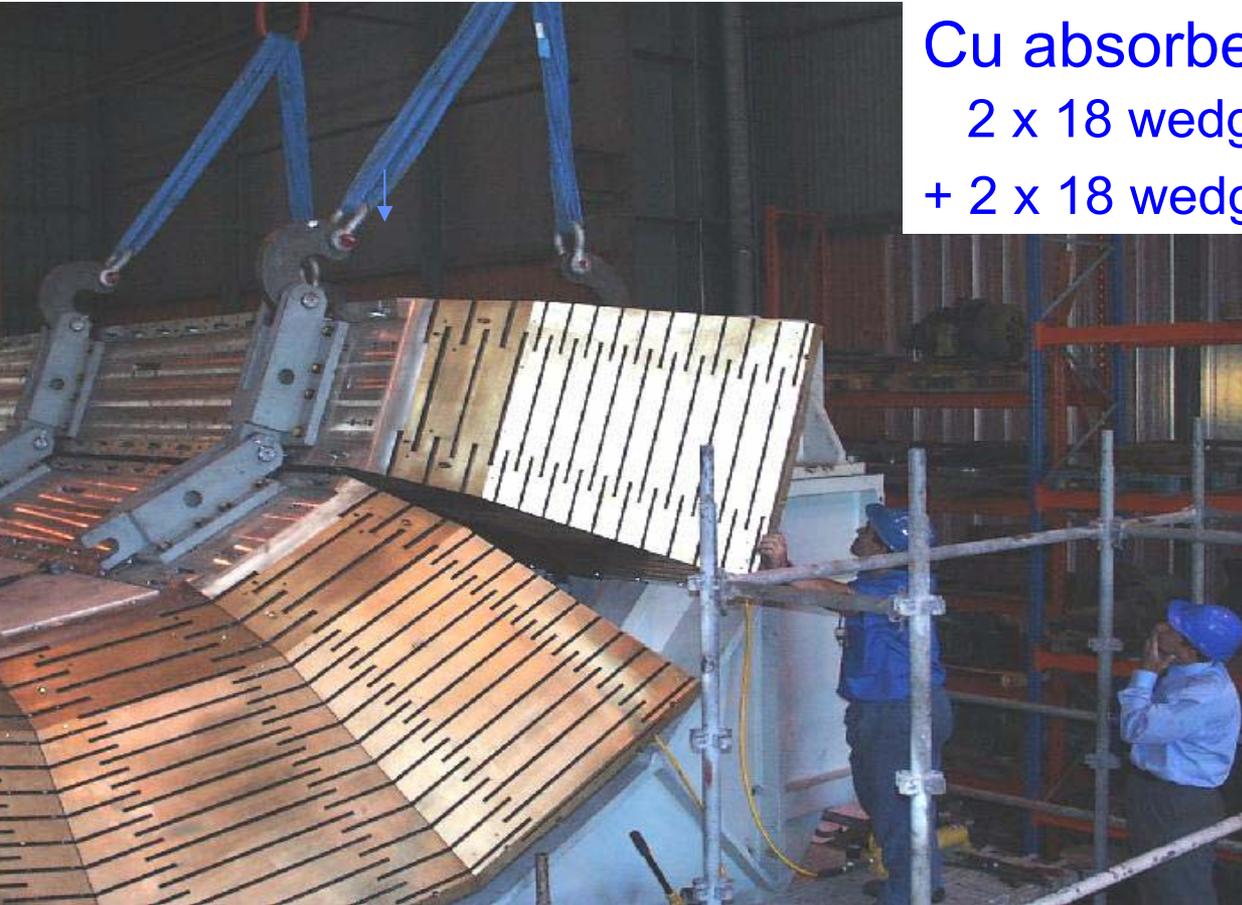
Sampling calorimeters

CMS Hadron calorimeter

Cu absorber + scintillators

2 x 18 wedges (barrel)

+ 2 x 18 wedges (endcap) \approx 1500 T absorber



Scintillators fill slots and are read out via fibres by HPDs

Best beam resolution for single hadrons

$$\frac{\sigma_E}{E} = \frac{65\%}{\sqrt{E}} \oplus 5\%$$