

# Does Quasi Neutrality Remain Valid in Pair-Ion Plasmas ?

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# Plan of the talk

1. Introduction
2. Pair-Ion (PI) Plasmas
3. Criterion to Determine (PI) Plasmas
4. Instability of A New Mode and Nonlinear Dynamics
5. Break Down of Quasi Neutrality
6. Conclusions

# 1. Introduction

Recently the efforts have been made to produce pure pair-ion fullerene ( $C_{60}^{\pm}$ ) plasmas in laboratories [1. Phys. Rev. Lett. 91 205005 (2003); 2. Phys. Rev. Lett. 95 , 175003 (2005)].

$$T_+ = T = 0.5 \text{ eV}$$

$$n_+ = n_- \sim 10^8 \text{ cm}^{-3}$$

$$B_0 \sim 0.3 \text{ T}$$

Three kinds of electrostatic waves, propagating only in the direction parallel to the external magnetic field have been discussed in the pure pair-ion plasma [2].

These are the ion plasma wave (IPW), the ion acoustic wave (IAW) and the third one has been named as the intermediate frequency wave (IFW) because it's frequency lies in between the frequencies of the other two waves i.e. the IPW and the IAW.

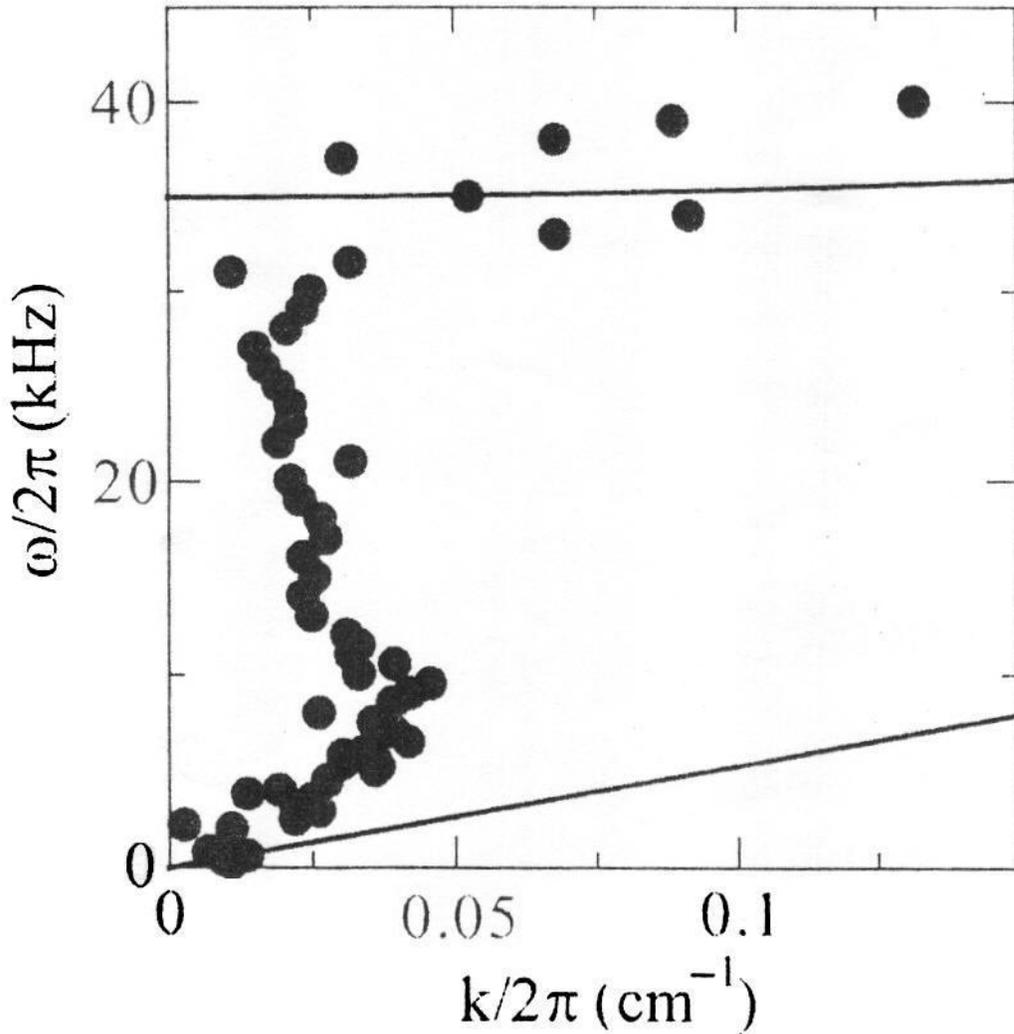


FIG. 1. Dispersion relations for electrostatic waves propagating along  $B$ -field lines. Solid lines and dots denote results calculated from two-fluid theory and measured experimentally, respectively.

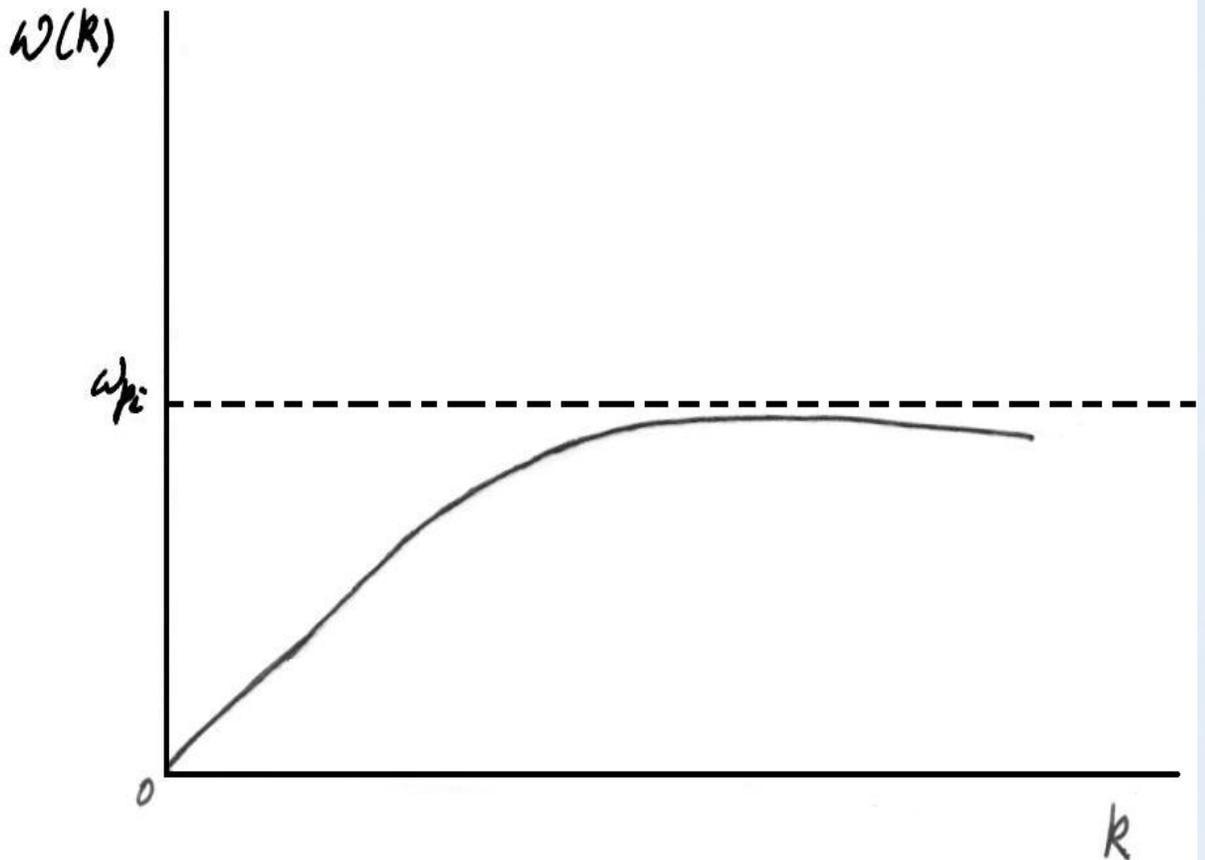


Fig 2: Wavenumber dependence of  $\omega$  on  $k$  for IAW in EI-Plasma.

$$\omega = \frac{c_s k}{(1 + \lambda_{De}^2 k^2)^{1/2}}$$

$$= \omega_{pi} \quad \text{for} \quad \lambda_{De}^2 k^2 \gg 1$$

$$\omega_{pi}^2 = \frac{c_s^2}{\lambda_{De}^2}$$

# Important Points

- A. It will be shown that the experimental observations itself indicate the existence of electrons in the system at a significant level. Therefore it does not seem to be a pure pair-Ion plasma.
- B. Quasi-neutrality is not a reasonable approximation in such plasmas, when these are perturbed.

# Definition of a Plasma

A quasi neutral statistical ensemble of charged particles which exhibits collective behavior due to long range electromagnetic forces. Neutral atoms (molecules) may also be present in the system.

Ex: H- Plasma

$$n_{0i} = H^+ \text{ cm}^{-3}; n_{0e} = e^- \text{ cm}^{-3};$$

$$n_{0i} = n_{0e} = n_0$$

Let:  $\lambda_{De}^2 = \left( \frac{\epsilon_0 T_e}{n_0 e^2} \right); \omega_{pe}^2 = \left( \frac{n_0 e^2}{\epsilon_0 m_e} \right)$

$\lambda_{De}$  Electron Debye length

$L$  System's dimension

$\nu$  Collision frequency of charged particles with neutrals

$\omega_{pe}$  Plasma oscillation frequency

## The Plasma demands

1)  $\nu \ll \omega_{pe}$  (i.e. em forces dominate)

2)  $\lambda_{De} \ll L$  (quasi neutrality)

3)  $1 \ll N_d = \left( \frac{4}{3} \pi \lambda_{De}^2 \right) n_0$

(statistical ensemble)

Quasi Neutrality  $n_{0e} = n_{0i}$  (Consider an EI-Plasma)

Perturbation  $E = -\nabla \phi$

$$\nabla \cdot \vec{E} = \frac{q}{\epsilon_0} (n_i - n_e) = -\nabla^2 \phi \quad 1.1$$

Assumption :  $|\lambda_{De}^2 \nabla^2| \ll 1$

For a linear wave  $|\nabla^2| \rightarrow |k^2|$

If  $\lambda_{De} \ll \text{wavelength}$

Then the long wavelength perturbation does not see the effects of charge separation. Therefore

may be used  $n_i = n_e$  1.2

But  $\varphi \neq 0$  or  $E \neq 0$  1.3

(1.2 together with 1.3) is the concept of quasi neutrality in plasmas.

## IAW in EI- plasmas:

i)  $\vec{B}_0 = B_0 \hat{z}$

ii) uniform plasma.

ion dynamics:

$$m_i n_i (\partial_t + \vec{v}_i \cdot \nabla) \vec{v}_i \quad 1.4$$
$$= e n_i (\vec{E} + \vec{v}_i \times B_0 \hat{z}) - \nabla p_i$$

$$\partial_t n_i + \nabla \cdot (n_i \vec{v}_i) = 0 \quad 1.5$$

Linearization:

Perturbation  $\propto \exp(i k_z z - \omega t)$  1.6

$$m_i n_0 \partial_t v_{iz} = e n_0 (-\partial_z \varphi) - \gamma_i T_i \partial_z n_i \quad 1.7$$

$$\partial_t n_i + n_0 \partial_z v_{iz} = 0 \quad 1.8$$

Electron dynamics :  $(m_e \rightarrow 0)$

$$n_e = n_0 e^{e\varphi/T_e} \approx n_0 (1 + e\varphi/T_e) \quad 1.9$$

Using  $n_i = n_e$  1.10

Eqs . (1.4--1.9) yield linear dispersion relation as,

$$\omega^2 = \left( \frac{T_e + \gamma_i T_i}{m_i} \right) k_z^2 \quad 1.11$$

where  $c_s = \left( \frac{T_e + \gamma_i T_i}{m_i} \right)^{1/2}$  is ion sound speed

For  $T_i \ll T_e; c_s^2 = \frac{T_e}{m_i}$

(1.11) becomes

$$\omega^2 = c_s^2 k_z^2 \quad (\text{Non dispersive IAW}) \quad 1.12$$

If  $\lambda_{De}^2 k^2 \neq 0$  we have

$$\omega^2 = \frac{c_s^2 k_z^2}{1 + \lambda_{De}^2 k^2} \quad (\text{Dispersive IAW}) \quad 1.13$$

$$\omega^2 = \omega_{pi}^2 \quad \text{for} \quad 1 \ll \lambda_{De}^2 k^2$$

# Electron-Ion and Pair Plasmas:

a. EI- Plasmas: Let  $\vec{\omega} = B_0 \hat{z}$

$$m_e \neq m_i; \Omega_i = \frac{eB_0}{m_i} \ll \Omega_e = \frac{eB_0}{m_e}$$

$$\omega_{pi} \ll \omega_{pe}$$

$$\lambda_i = \frac{c}{\omega_{pi}} \gg \lambda_e = \frac{c}{\omega_{pe}}$$

$$v_{Te}^2 = \frac{T_e}{m_e}; v_{Ti}^2 = \frac{T_i}{m_i}$$

$\lambda_{Di} \neq \lambda_{De}$  Q ( $T_e \neq T_i$ ) in general

Slow time scales  $|\partial_t|$  or  $\omega \ll \Omega_i$

Fast time scales  $\partial_t \sim \Omega_e, \omega_{pe}$

Hybrid time scale  $\omega_{pi}, \Omega < \omega < \Omega_e, \omega_{pe}$

**b. Pair plasmas**  $m_+ = m_-$ , No diff in time scales. Ion acoustic wave does not exist in such plasmas.

Examples : Electron-Positron Plasmas in

- i) Pulsar Magnetospheres
- ii) Active Galactic Nuclie (AGN)
- iii) Laboratory

Pair-Ion plasmas:

- i) Recent claims that pure pair-ion fullerene plasmas have been produced in laboratories(?)

## 2. Pair-Ion Plasmas

Let  $\vec{\omega} = B_0 \hat{z} \equiv \text{constant}$ ,  $\vec{E} = -\nabla \phi$  and consider the plasma to be homogeneous. [3.Phys.Lett.A 350,375(2006)]. For  $\alpha$ -species

$$m_\alpha n_\alpha \partial_t \vec{v}_\alpha = n_\alpha q_\alpha (\vec{E} + \vec{v}_\alpha \times B_0 \hat{z}) - \nabla p_\alpha, \quad 2.1$$

$$(\partial_t^2 + \Omega_\alpha^2) \vec{v}_{\alpha\perp} = \frac{q_\alpha}{m_\alpha} (\partial_t \vec{E}_\perp + \Omega_\alpha \vec{E}_\perp \times \hat{z}) \quad 2.2$$

$$-\frac{\Omega_\alpha}{m_\alpha} \frac{\nabla_\perp p_\alpha \times \hat{z}}{n_\alpha} - \partial_t \frac{\nabla_\perp p_\alpha}{m_\alpha n_\alpha}$$

$$\partial_t v_{\alpha z} = \frac{q_\alpha}{m_\alpha} E_z - \frac{\partial_z p_\alpha}{m_\alpha n_\alpha} \quad 2.3$$

$$\partial_t n_\alpha + n_{0\alpha} \nabla_\perp \cdot \vec{v}_{\alpha\perp} + n_{0\alpha} \partial_z v_{\alpha z} = 0 \quad 2.4$$

Eqs. (2.2-2.4) give,

$$\begin{aligned} & \{ \omega^2 (\omega^2 - \Omega_\alpha^2) - v_{T\alpha}^2 k_\perp^2 \omega^2 + v_{T\alpha}^2 k_z^2 \Omega_\alpha^2 \} n_\alpha \\ & - \frac{n_{0\alpha} q_\alpha}{m_\alpha} k_\perp^2 \omega^2 \varphi - \frac{n_{0\alpha} q_\alpha}{m_\alpha} (\omega^2 - \Omega_\alpha^2) k_z^2 \varphi = 0 \end{aligned} \quad 2.5$$

Writing Eq. (2.5) for  $\alpha = \pm$  and then subtracting one equation from the other, we obtain

$$\begin{aligned} & [ \omega^2 (\omega^2 - \Omega_i^2) - v_{Ti}^2 k_\perp^2 \omega^2 + v_{Ti}^2 k_z^2 \Omega_i^2 ] (n_+ - n_-) \\ & - (n_+^0 + n_-^0) \frac{q}{m_i} k_\perp^2 \omega^2 \varphi \\ & - (n_+^0 + n_-^0) \frac{q}{m_i} k_z^2 (\omega^2 - \Omega_i^2) \varphi = 0 \end{aligned} \quad 2.6$$

where  $\Omega_i = qB_0/m_i$  is the ion gyro frequency.

Also  $q_+ = q_- = q$  and  $m_+ = m_- = m_i$  has been assumed

Let  $T_+ = T_- = T_i$ ,  $v_{Ti} = (\gamma_i T_i / m_i)^{1/2}$  and  $\gamma_i$  be the ratio of specific heats. The Poisson equation reads

$$(n_+ - n_-) = -\frac{\epsilon_0}{q} \nabla^2 \phi. \quad 2.7$$

Let us assume for the time being that electrons are also present in the system and they obey the Boltzmann density distribution,

$$n_e = n_{0e} \exp\left(\frac{e\phi}{T_e}\right) \quad 2.8$$

The set of equations (2.5--2.7) yields a few simple but interesting results. Let us discuss the limiting cases one by one. We observe that a new mode which may be called a finite frequency pair plasma convective cell (PPCC) can exist in such systems in the quasi neutral approximation. Let's assume  $n_+ \approx n_-$ , then Eq.(2.6) with  $n_e \approx 0$  and  $\omega \ll \Omega_i$  gives,

$$\omega^2 = \frac{k_z^2}{k_\perp^2} \Omega_i^2 \quad 2.9$$

Note:

In dusty plasmas a similar mode has been investigated. It may exist due to the presence of stationary dust in EI plasmas.

Dispersion relation is

$$\omega = \frac{n_{0e}}{n_{0i}} \frac{m_i}{m_e} \frac{k_z}{k_{\perp}} \Omega_i \quad 2.10$$

### 3. Criterion to Determine a PI Plasma

IAW can not be observed in pure PI plasma. We

want to find out a criterion to determine  $\frac{n_{0e}}{n_+^0}$ .

Let  $n_+ \cong n_-, k_\perp = 0$ , then for  $\omega^2 \ll \Omega_i^2$

Eqs.(2.6) & (2.8) yield,

$$\omega^2 = \frac{q}{e} (N_0 c_s^2 k_z^2) + v_{Ti}^2 k_z^2 \quad 3.1$$

where 
$$N_0 = \frac{n_+^0 + n_-^0}{n_{0e}} = \frac{1 + \epsilon}{1 - \epsilon}$$

And 
$$\epsilon = \frac{n_-^0}{n_+^0}.$$

If  $1 \ll N_0$  the frequency of

the wave may become larger than  $c_s^2 k_z^2$ .

We may have also  $\Omega_i < \omega$  even if  $c_s k_z < \Omega_i$ . The observation of IAW indicates the presence of electrons in the system. Therefore we expect

$$n_{0e} \neq 0$$

$$c_s k_z < \omega_s \left( \omega_s \neq c_s k_z \ominus n_-^0 \neq 0 \right)$$

in the experiment as the plot of acoustic wave in Fig.2 of [2] shows (our Fig 1). Since  $c_s^2 k^2 < \omega^2$  holds if  $1 < N_0$ , therefore we can determine  $n_{0e}$  by measuring the frequency of IAW.

If  $T_i = 0, k_{\perp} \neq 0$  and  $n_+ \approx n_-$ , Eqs. (2.6) and (2.8) yield,

$$\begin{aligned} \omega^4 - \left( \Omega_i^2 + \frac{q}{e} N_0 c_s^2 k^2 \right) \omega^2 \\ + \frac{q}{e} N_0 c_s^2 k_z^2 \Omega_i^2 = 0 \end{aligned} \quad 3.2$$

In the electron ion plasma case  $N_0=1$  and for the ion cyclotron wave we have  $k_z \ll k_\perp$ , therefore Eq.(3.2) yields the well known dispersion relation  $\omega^2 = \Omega_i^2 + c_s^2 k^2$ . In the present situation,  $1 \ll N_0$  is possible along with  $\omega^2 < N_0 c_s^2 k_z^2$ , therefore we retain the last term in Eq.(3.2). It gives for  $q = e$ ,

$$\omega^2 = \frac{1}{2} [ (\Omega_i^2 + N_0 c_s^2 k^2) \pm (\Omega_i^2 + N_0 c_s^2 k^2)^2 - 4 N_0 c_s^2 k_z^2 \Omega_i^2 ]^{1/2} \quad 3.3$$

This is the modified ion cyclotron wave dispersion relation. In the limit  $\omega \ll \Omega_i$  it reduces to the ion acoustic wave:

$$\omega^2 = \frac{q N_0 c_s^2 k_z^2 / e}{1 + q N_0 \rho_s^2 k^2 / e} \quad 3.4$$

$$\rho_s^2 = \frac{c_s^2}{\Omega_i^2}$$

This is obliquely propagating IAW. In our opinion, (6.3) gives the so called IMF wave of Ref [2]. But  $k_\perp$  should also be measured along with  $T_e$ .

## 4. Instability of A New Mode and Nonlinear Dynamics

Let there be an external electric field  $\vec{E}_0 = -\vec{\nabla}\phi_0$  and let the plasma flow be along  $y$ - direction such that both positive and negative ions move with the same shear velocity, *i.e.*  $v_+(x) = v_-(x) = v_0(x)$  and, hence the background current is zero. The steady state demands  $\vec{E}_0 = -\mathbf{B}_0 v_0$ .

For  $|\partial_t| < \Omega_i$  Eqs.(2.1) gives,

$$\begin{aligned} \vec{v}_{\alpha\perp} &= \frac{1}{B_0} (\vec{E}_\perp \times \hat{z}) - \frac{1}{\Omega_\alpha} (\partial_t + \vec{v}_\alpha \cdot \vec{\nabla}) \vec{v}_\alpha \times \hat{z} \\ &= \vec{v}_E + \vec{v}_{\alpha p} \end{aligned} \quad 4.1$$

and

$$(\partial_t + \mathbf{v}_\alpha \cdot \nabla) \mathbf{v}_{\alpha z} = \frac{q_\alpha}{m_\alpha} \mathbf{E}_z \quad 4.2$$

Note :

$$\nabla \cdot (\mathbf{v}_{+p} - \mathbf{v}_{-p}) = -\frac{2}{B_0 \Omega_i} [\partial_t +$$

$$\frac{1}{B_0} \mathbf{z} \times \nabla_\perp (\varphi + \varphi_0) \cdot \nabla_\perp] \nabla_\perp^2 (\varphi + \varphi_0)$$

Then (4.2) yields

$$\left[ \frac{\partial}{\partial t} + \frac{1}{B_0} \mathbf{z} \times \nabla_\perp (\varphi + \varphi_0) \cdot \nabla_\perp \right]$$

$$(\mathbf{v}_{+z} - \mathbf{v}_{-z}) = -b \frac{\partial \varphi}{\partial z}, \quad 4.3$$

and continuity Eqs. yield the nonlinear Eq.

$$\left[ \frac{\partial}{\partial t} + \frac{1}{B_0} \mathbf{r} \times \nabla_{\perp} (\varphi + \varphi_0) \cdot \nabla_{\perp} \right] \quad 4.4$$

$$a \nabla_{\perp}^2 (\varphi + \varphi_0) = - \frac{\partial}{\partial z} (v_{+z} - v_{-z}),$$

where  $a = -2 / (B_0 \Omega)$  and  $b = 2q / mi$

Assuming the linear perturbation of the form  $\varphi(x) \exp i(k_y y + k_z z - \omega t)$ , we obtain from above Eqs.

$$\frac{d^2 \varphi}{dx^2} - k_y^2 \varphi + \frac{k_y v_0''}{\omega - v_0 k_y} \varphi \quad 4.5$$

$$- \frac{k_z \Omega^2}{(\omega - v_0 k_y)^2} \varphi = 0,$$

where the superscript double prime indicates the second-order differentiation with respect to  $x$ . Let  $\omega = \omega_r + i\gamma$ , where  $\omega_r$  and  $\gamma$  denote the real and imaginary parts of the frequency, respectively.

Then the instability condition turns out to be

$$\frac{k_y v_0''}{\omega_0^2 + \gamma^2} + \frac{2k_z^2 \Omega_i^2 \omega_2}{\omega_0^4 + 4\gamma^2 \omega_0^2} = 0, \quad 4.6$$

where

$$\omega_0 = (\omega_r - v_0 k_y)$$

Let us consider the nonlinear stage of instability in a frame which is moving with velocity components  $U_y$  and  $U_z$  in the  $yz$  – plane with  $\alpha = U_y / U_z$ . Eq.

(7.3) can be written in this frame as

$$[e_z \times \nabla_{\perp} (\Phi - B_0 U_y x) \cdot \nabla_{\perp}] \quad 4.7$$

$$[(v_{+z} - v_{-z}) - B_0 \alpha b x] = 0,$$

where  $\Phi = \varphi + \varphi_0$ . Then the solution of Eq. (4.3) becomes

$$(v_{+z} - v_{-z}) - B_0 \alpha b x = F(\Phi - B_0 U_y x) \quad 4.8$$

where  $F$  is an arbitrary function of the given argument. We choose a linear form of this function  $F(\Phi - B_0 U_y x) = F_0 (\Phi - B_0 U_y x)$  with  $F_0$  an arbitrary constant.

Thus the above equation becomes

$$(v_{+z} - v_{-z}) = F_0 \Phi + B_0 g x \quad 4.9$$

where  $g = (\alpha b - F_0 U_y)$ . Similarly, Eq. (4.4) can be written as

$$\nabla_{\perp}(\Phi - B_0 U_y x) \times \nabla_{\perp}(a \nabla^2 \Phi - F_0 B_0 \alpha x) = 0, \quad 4.10$$

which yields

$$\begin{aligned} \nabla_{\perp}^2 \Phi - G_0 \Phi + (G_0 B_0 U_y \\ - \frac{F_0}{\alpha} B_0 \alpha) = 0, \end{aligned} \quad 4.11$$

where  $G_0$  is a constant. It is well known that equation (4.11) admits dipolar and tricolor vortex solutions.

## 5. Break down of quasi neutrality in PI plasmas.

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} (q_+ n_+ - q_- n_- - e n_e) \quad 5.1$$

Let  $q_+ = q_- = e$  and  $\mathbf{E} = -\nabla \phi$

$$-\nabla^2 \phi = \frac{e}{\epsilon_0} (n_+ - n_- - n_e) \quad 5.2$$

$$-\lambda_{De}^2 \nabla^2 \left( \frac{e\phi}{T_e} \right) = \left( \frac{n_+}{n_{0e}} - \frac{n_-}{n_{0e}} - \frac{n_e}{n_{0e}} \right)$$

It  $n_- = 0$ , then  $n_e = n_{e0} e^{e\phi/T_e}$  is assumed and for  $n_+$  ion continuity eq is used. then quasi neutrality ( $n_e \sim n_i$ ) yields (for  $k_{\perp} = 0$ ),

$$\omega_s^2 = c_s^2 k_z^2 \quad 5.3$$

Since  $\lambda_{De}^2 k^2 \ll 1$  for low frequency IAW, therefore quasi neutrality is valid in general. If  $\lambda_{De}^2 k^2 < 1$ , then the above dispersion relation becomes,

$$\omega^2 = \frac{c_s^2 k_z^2}{1 + \lambda_{De}^2 k^2} \quad 5.4$$

For  $1 \ll \lambda_{De}^2 k^2$ , and  $k_{\perp} = 0$

(short wavelength limit), (5.4) becomes

$$\omega^2 = \omega_{pi}^2 \quad 5.5$$

Note :  $\lambda_{De}^2 = \frac{\epsilon_0 T_e}{n_{0e} e^2}$ .

Small  $n_{0e}$  implies large  $\lambda_{De}$ .

If  $n_- \neq 0$  and  $n_- \approx n_+$ , then for  $n_{0e} \ll n_+$  the situation  $1 < \lambda_{De}^2 k^2$  seems to be very common in Pair-ion-electron (pie) plasmas.

The real freq  $\omega_r$  in pie plasmas becomes,

$$\omega_r(k) = \omega_s \left( 1 + \frac{3}{2} \frac{v_{Ti}^2 k^2}{\omega_s^2} \right) \quad 5.6$$

where

$$\omega_s = N_0 \frac{c_s^2 k^2}{1 + \lambda_{De}^2 k^2} \quad 5.7$$

Using kinetic model for  $T_i \neq 0$ , the dispersion relation for IAW can be written as,

$$1 + \sum_j \frac{1}{k^2 \lambda_{Dj}^2} \{ 1 + i \sqrt{\pi} w(z_j) \} = 0 \quad 5.8$$

where

$$z_j = \frac{\omega}{\sqrt{2k}v_{Tj}} : \lambda_{Dj} = \frac{v_{Tj}}{\omega_{pj}},$$

$$\omega_{pj} = \left( \frac{4\pi n_{0j} q^2}{m_j} \right)^{1/2}, v_{Tj} = \frac{T_j}{m_j}$$

Here  $W(z_j)$  is called plasma dispersion function

$$W(z_j) \approx 1 \quad \text{if} \quad |z_j| \ll 1 \quad 5.9a$$

$$W(z_j) = \frac{i}{\sqrt{\pi}z_j} \left( 1 + \frac{1}{2z_j^2} + \frac{3}{4z_j^4} \right) + \exp(-z_j^2) \quad 5.9b$$

For  $1 \ll |z_j|$ .

For analytical analysis of IAW, let us assume,

$$v_{\pm} \ll \frac{\omega}{k} \ll v_{Te} \quad 5.10$$

If  $\lambda_{De}^2 k^2 \ll 1 \Rightarrow \omega^2 = N_0 c_s^2 k^2 + 3 v_{Ti}^2 k^2$

If  $1 \ll \lambda_{De}^2 k^2$  for  $n_{0e} \ll n_0^+$ , then we have for

$$\omega = \omega_r - i\gamma$$

$$\omega_r(k) \approx (1 + \epsilon) \omega_{pi}^+ \tag{5.11}$$

$$\left[ \left(1 - \frac{1}{2 \lambda_{De}^2 k^2}\right) + \frac{3}{2} \frac{n_+^0}{n_{0e}} \frac{\lambda_{D+}^2 k^2}{N_0} \right]$$

where  $\epsilon = \frac{n_-^0}{n_+^0}$  and  $\omega_{pi}^+ = \frac{n_+^0 e^2}{\epsilon_0 m_i}$ . Eq. (5.11)  $\Rightarrow$

$$\omega_{pi} < \omega_r \quad (\text{may hold}) \tag{5.12}$$

in pie plasmas for  $n_{0e} \neq 0$

Let

$$\lambda_{De}^2 = \epsilon_1 \lambda_{De+}^2 = \frac{n_+^0}{n_-^0} \left( \frac{\epsilon_0 T_e}{n_+^0 e^2} \right) \tag{5.13}$$

For example if  $n_0^- = 0.9n_0^+, N_0 = 19$

Then  $\lambda_{De}^2 = 10\lambda_{De+}^2$ . If  $\lambda_{De+}^2 k^2 = 10$  is assumed

(which is equivalent to  $\lambda_{De+}^2 k^2 = 10$  in case of ei plasmas), then in pie

plasmas, we find  $\epsilon_1 = 10$  and hence

$$\lambda_{De}^2 k^2 = 10^2 \gg 1$$

Imaginary part of the frequency can be written as,

$$\gamma = \gamma_e^\pm + \gamma_i^\pm \quad 5.14$$

where

$$\gamma_e^\pm = N_0 \gamma_e \quad 5.15$$

$$\gamma_i^\pm = N_0^2 \gamma_i \quad 5.16$$

$$\gamma_e(k) \approx \left(\frac{\pi}{8}\right)^{1/2} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{c_s k}{(1 + \lambda_{De}^2 k^2)^2} \quad 5.17$$

$$\gamma_i(k) \approx \left(\frac{\pi}{8}\right)^{1/2} \left(\frac{T_e}{T_i}\right)^{3/2} \frac{c_s k}{(1 + \lambda_{Dc}^2 k^2)^2} \times \exp\left(-\frac{\omega_\gamma^2(k)}{2k^2 v_{Ti}^2}\right) \quad 5.18$$

For  $n_-^0 = 0$  above relation reduce to the case of ei plasmas where  $N_0 = 1$  [5. Akhiezer et. al. 1975]

In the limit  $1 \ll \lambda_{De}^2 k^2$  we obtain

$$\gamma_e^\pm(k) \approx \left[ \sqrt{\frac{\pi}{8}} \sqrt{\frac{m_e}{m_i}} \frac{1 - \epsilon^2}{\lambda_{D+}^2 k^2} \right] c_s k \quad 5.19$$

and

$$\gamma_i^\pm(k) \approx \left[ \sqrt{\frac{\pi}{8}} \left(\frac{T_e}{T_i}\right)^{3/2} \frac{(1 + \epsilon)^2}{(\lambda_{D+}^2 k^2)^2} \exp\left(-\frac{\omega_\gamma^2}{2k^2 k_{Ti}^2}\right) \right] c_s k \quad 5.20$$

# 6. Conclusion

## I. Damping analysis:

$$\exp\left(-\frac{\omega_r^2}{2k^2 v_{Ti}^2}\right) \ll 1$$

and  $\epsilon \leq 1$  or  $\epsilon \ll 1$

Therefore  $\gamma_i^\pm \leq \gamma_i$

For the same  $\lambda_{De+}^2 k^2$  in ei and pie plasmas, we find

i)  $\gamma_i^\pm < \gamma_i$

ii)  $\gamma_e^\pm < \gamma_e$  always because  $(1 - \epsilon^2) < 1$

So  $\gamma^\pm < \gamma$  for  $1 \ll \lambda_{De}^2 k^2$ . 6.1

II.  $1 \ll \lambda_{De}^2 k$  may hold because  $1 \ll \epsilon_1$

even for  $\lambda_{De}^2 k \leq 1$ . The quasi neutrality is not a good approximation in such systems.

Therefore IAW can be easily excited in pair-ion plasmas comprising electrons. This can be a possible explanation for the density fluctuation associated with IAW in experiment [ 2]. Hence these plasmas do not seem to be the pure ion plasmas.

III. A new mode may be interesting in pair plasmas. It can also become unstable in certain situations.

IV.  $c_s k \ll \omega_r$  for IAW in pair-ion plasmas having electrons. Measuring  $T_e$  and  $c_s k$ , one

can estimate  $\frac{n_{0e}}{n_+^0}$ .

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