

# CP-Violation in B Decays and Final State Strong Phases

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# Introduction



**Symmetry's handiwork.** The left and right human hands cannot be superimposed by simple rotations and translations; they are mirror images of each other. [Woodcut by Albrecht Dürer (1471–1528), from the Bettmann Archive]

Figure: Chirality

# Introduction

- In weak interaction, both P and C are violated but CP is conserved by the weak interaction Lagrangian. Hence for  $X^0 - \bar{X}^0$  complex ( $X^0 = K^0, B^0, B_s^0$ ); the mass matrix is not diagonal in  $|X^0\rangle$  and  $|\bar{X}^0\rangle$  basis.
- However, assuming CP conservation, the CP eigenstates  $|X_1^0\rangle$  and  $|X_2^0\rangle$  can be mass eigenstates and hence mass matrix is diagonal in this basis.
- The two sets of states are related to each other by superposition principle of quantum mechanics.
- This gives rise to quantum mechanical interference so that even if we start with a state  $|X^0\rangle$ , the time evolution of this state can generate the state  $|\bar{X}^0\rangle$ . This is a source of mixing induced CP violation.
- However, both in  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  complex, the mass eigenstates  $|K_S^0\rangle, |K_L^0\rangle$  and  $|B_L^0\rangle, |B_H^0\rangle$  are not CP eigenstates.
- In the case of  $K^0 - \bar{K}^0$  complex, there is a small admixture of wrong CP state characterized by a small parameter  $\epsilon$ , which gives rise to the CP violating decay  $K_L^0 \rightarrow \pi^+ \pi^-$ . This was the first CP violating decay observed experimentally.

- For  $B^0 - \bar{B}^0$  complex, the mismatch between mass eigenstates and  $CP$  eigenstates  $|B_1^0\rangle$  and  $|B_2^0\rangle$  is given by the phase factor  $e^{2i\phi_M}$  where the phase factor  $\phi_M = -\beta$  in the standard model viz. one of the phases in the CKM matrix.
- For  $B_s^0 - \bar{B}_s^0$ , there is no mismatch between  $CP$  eigenstates  $|B_{1s}^0\rangle$  and  $|B_{2s}^0\rangle$  and the mass eigenstates.
- There is no extra phase available in CKM matrix, with three generations of quarks to accommodate more than two independent phases  $\beta$  and  $\gamma$ ; the unitarity of CKM matrix requires  $\alpha + \beta + \gamma = \pi$ .
- The quantum mechanical interference gives rise to non zero mass differences  $\Delta m_K$ ,  $\Delta m_B$  and  $\Delta m_{B_s}$  between mass eigenstates. The mixing induced  $CP$  violation involves these mass differences.
- The  $CPT$  invariance plays an important role in  $CP$  violation in weak decays.  $CPT$  invariance gives

$$\bar{A}_{\bar{f}} = \eta_f e^{2i\delta_f} A_f^*, \quad A_f = e^{i\delta_f} e^{i\phi} |A_f| = e^{i\phi} F_f$$

where  $A_f$  and  $\bar{A}_{\bar{f}}$  are the amplitudes for the decays  $X \rightarrow f$  and  $\bar{X} \rightarrow \bar{f}$ , the states  $|f\rangle$  and  $|\bar{f}\rangle$  being  $CP$  conjugate of each other.

- For direct  $CP$  violation, at least two amplitudes with different weak phase are required:

$$A_f = A_{1f} + A_{2f}$$

$CPT$  gives:

$$\begin{aligned}\bar{A}_{\bar{f}} &= e^{2i\delta_{1f}} A_{1f}^* + e^{2i\delta_{2f}} A_{2f}^* \\ A_{if} &= e^{i\delta_{if}} e^{i\phi_i} |A_{if}| = e^{i\phi_i} F_{if}\end{aligned}$$

where  $(\delta_{1f}, \delta_{2f})$ ,  $(\phi_1, \phi_2)$  are strong final state phases and the weak phases respectively.

- Thus the direct  $CP$  violation is given by

$$A_{CP} = \frac{\bar{\Gamma}(\bar{X} \rightarrow \bar{f}) - \Gamma(X \rightarrow f)}{\bar{\Gamma}(\bar{X} \rightarrow \bar{f}) + \Gamma(X \rightarrow f)} = \frac{2|A_{1f}||A_{2f}| \sin \delta_f \sin \phi}{|A_{1f}|^2 + |A_{2f}|^2 + 2 \cos \delta_f \cos \phi}$$

where  $\delta_f = \delta_{2f} - \delta_{1f}$ ,  $\phi = \phi_2 - \phi_1$ . Hence the necessary condition for non-zero direct  $CP$  violation is  $\delta_f \neq 0$  and  $\phi \neq 0$ .

- We note the *CKM* matrix  $V$  relates the weak eigenstates  $d', s'$  and  $b'$  with the mass eigenstates  $d, s$  and  $b$ .

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1)$$

- Thus the strong interactions play a crucial role in the *CP*-asymmetries.
- The short distance strong interactions effects at quark level are taken care of by perturbative QCD in terms of Wilson coefficients.
- The CKM matrix, which connects the weak eigenstates with mass eigenstates, is another aspect of strong interactions at quark level.
- In the case of semi leptonic decays, the long distance strong interaction effects manifest themselves in the form factors of final states after hadronization. Likewise the strong interaction final state phases are long distance effects.
- Strong phase shifts essentially arise in terms of S-matrix which changes an 'in' state into an 'out' state viz.

$$|f\rangle_{in} = S|f\rangle_{out} = e^{2i\delta_f}|f\rangle_{out} \quad (2)$$

- It is difficult to reliably estimate the final state strong phase shifts. It involves the hadronic dynamics. However, using isospin, C-invariance of S-matrix and unitarity, we can relate these phases.
- In this regard, following cases are of interest:
 

**Case (I):** The decays  $B^0 \rightarrow f, \bar{f}$  described by two independent single amplitudes  $A_f$  and  $A'_{\bar{f}}$  with different weak phases:

$$A_f = \langle f | \mathcal{L}_{\mathcal{W}} | B^0 \rangle = e^{i\phi} F_f = e^{i\phi} e^{i\delta_f} |F_f|$$

$$A'_{\bar{f}} = \langle \bar{f} | \mathcal{L}'_{\mathcal{W}} | B^0 \rangle = e^{i\phi'} F'_{\bar{f}} = e^{i\phi'} e^{i\delta'_{\bar{f}}} |F'_{\bar{f}}|$$

where the states  $|\bar{f}\rangle$  and  $|f\rangle$  are C conjugate of each other such as states  $D^{(*)-} \pi^+ (D^{(*)+} \pi^-)$ ,  $D_s^{(*)-} K^+ (D_s^{(*)+} K^-)$ ,  $D^- \rho^+ (D^+ \rho^-)$ .

For this case, there is an added advantage that the decay amplitudes  $A_f$  and  $A'_{\bar{f}}$  are given by tree graphs. Assuming factorization for tree graphs, one can get information for the  $B \rightarrow D^{(*)}$  form factors.

**Case (II):** The weak amplitudes  $A_f \neq A'_{\bar{f}}$ ,

$$A_f = \langle f | \mathcal{L}_{\mathcal{W}} | B^0 \rangle = [e^{i\phi_1} F_{1f} + e^{i\phi_2} F_{2f}]$$

$$A'_{\bar{f}} = \langle \bar{f} | \mathcal{L}_{\mathcal{W}} | B^0 \rangle = [e^{i\phi_1} F_{1\bar{f}} + e^{i\phi_2} F_{2\bar{f}}]$$

as is the case for the following decays,

$$B^0 \rightarrow \rho^- \pi^+(f) : A_f, \quad B^0 \rightarrow \rho^+ \pi^-(\bar{f}) : A_{\bar{f}}$$

- The  $C$ - invariance of S-matrix gives  $S_{\bar{f}} = S_f$  which implies

$$\delta_f = \delta'_{\bar{f}}, \quad \delta_{1f} = \delta_{1\bar{f}}, \quad \delta_{2f} = \delta_{2\bar{f}}$$

# Unitarity and Final State Strong Phases

The time reversal invariance gives

$$F_f =_{out} \langle f | \mathcal{L}_W | B \rangle =_{in} \langle f | \mathcal{L}_W | B \rangle^* \quad (3)$$

where  $\mathcal{L}_W$  is the weak interaction Lagrangian without the CKM factor such as  $V_{ud}^* V_{ub}$ . From Eq. (3), we have

$$\begin{aligned} F_f^* &=_{out} \langle f | S^\dagger \mathcal{L}_W | B \rangle \\ &= \sum_n S_{nf}^* F_n \end{aligned} \quad (4)$$

It is understood that the unitarity equation which follows from time reversal invariance holds for each amplitude with the same weak phase.

Above equation can be written in two equivalent forms:

① Exclusive version of Unitarity

Writing

$$S_{nf} = \delta_{nf} + iM_{nf} \quad (5)$$

we get from Eq (4) ,

$$\text{Im}F_f = \frac{1}{2} \sum_n M_{nf}^* F_n \quad (6)$$

where  $M_{nf}$  is the scattering amplitude for  $f \rightarrow n$  and  $F_n$  is the decay amplitude for  $B \rightarrow n$ . In this version, the sum is over all allowed exclusive channels. This version is more suitable in a situation where a single exclusive channel is dominant one. To get the final result, one uses the dispersion relation.

② Inclusive version of Unitarity

This version is more suitable for our analysis. For this case, we write Eq. (4) in the form

$$F_f^* - S_{ff}^* F_f = \sum_{n \neq f} S_{nf}^* F_n \quad (7)$$

Parametrizing S-matrix as  $S_{ff} \equiv S = \eta e^{2i\Delta}$ ,  $0 \leq \eta \leq 1$ , we get after taking the absolute square of both sides of Eq.(7)

$$|F|^2 [(1 + \eta^2) - 2\eta \cos 2(\delta_f - \Delta)] = \sum_{n,n'} F_n S_{nf}^* F_{n'}^* S_{n'f} \quad (8)$$

The above equation is an exact equation. In the random phase approximation of Wolfenstein and Suzuki, we can put

$$\begin{aligned} \sum_{n', n' \neq f} F_n S_{nf}^* F_{n'} S_{n'f} &= \sum_{n \neq f} |F_n|^2 |S_{nf}|^2 \\ &= |\bar{F}_n|^2 (1 - \eta^2) \end{aligned} \quad (9)$$

We note that in a single channel description:

$$(Flux)_{in} - (Flux)_{out} = 1 - |\eta e^{2i\Delta}|^2 = 1 - \eta^2 = \text{Absorption}$$

The absorption takes care of all the inelastic channels.

Similarly for the amplitude  $F_{\bar{f}}$ , we have

$$F_{\bar{f}}^* - S_{\bar{f}\bar{f}}^* F_{\bar{f}} = \sum_{\bar{n} \neq \bar{f}} S_{\bar{n}\bar{f}}^* F_{\bar{n}} \quad (10)$$

The C-invariance of S-matrix gives:

$$\begin{aligned} S_{fn} &= \langle f|S|n\rangle = \langle f|C^{-1}CSC^{-1}C|n\rangle \\ &= \langle \bar{f}|S|\bar{n}\rangle = S_{\bar{f}\bar{n}} \end{aligned} \quad (11)$$

Thus in particular C-invariance of S-matrix gives

$$S_{\bar{f}\bar{f}} = S_{ff} = \eta e^{2i\Delta} \quad (12)$$

Hence from Eq. (12), using Eqs. (9 – 12), we get

$$\frac{1}{1 - \eta^2} [(1 + \eta^2) - 2\eta \cos 2(\delta_{f\bar{f}} - \Delta)] = \rho^2, \bar{\rho}^2 \quad (13)$$

where

$$\rho^2 = \frac{|\overline{F_n}|^2}{|F_f|^2}, \quad \bar{\rho}^2 = \frac{|\overline{F_{\bar{n}}}|^2}{|F_{\bar{f}}|^2} \quad (14)$$

From Eq.(13), we get

$$\sin(\delta_{f\bar{f}} - \Delta) = \pm \sqrt{\frac{1 - \eta^2}{4\eta}} \left[ \rho^2, \bar{\rho} - \frac{1 - \eta}{1 + \eta} \right]^{1/2} \quad (15)$$

The maximum value for  $\rho^2, \bar{\rho}^2$  is 1 and the minimum value for them is  $\frac{1-\eta}{1+\eta}$ . Hence we get the following bounds:

$$\begin{aligned} \frac{1-\eta}{1+\eta} &\leq \rho^2, \bar{\rho}^2 \leq 1 \\ 0 &\leq \delta_{f,\bar{f}} - \Delta \leq \theta \\ -\theta &\leq \delta_f - \Delta \leq 0 \end{aligned} \tag{16}$$

$$\theta = \sin^{-1} \sqrt{\frac{1-\eta}{2}} \tag{17}$$

From now on, we will confine our self to positive square root in Eq(15).

The strong interaction parameter  $\Delta$  and  $\eta$  in the above bounds can be obtained by strong interaction dynamics. Using  $SU(2)$ , C-invariance of strong interaction and Regge phenomonology, the scattering amplitude  $M(s, t)$  for two particle final state can be determined.[See details Fayyazuddin PhysRevD.80.094015]. The  $s$ -wave scattering amplitude  $f$  is given by

$$f \approx \frac{1}{16\pi s} \int_{-s}^0 M(s, t) dt, t \approx -\frac{1}{2}s(1 - \cos \theta) \tag{18}$$

Using the relation  $S = \eta e^{2i\Delta} = 1 + 2if$ , where  $f$  is given by Eq.(18), the phase shift  $\Delta$ , the parameter  $\eta$  and the phase angle  $\theta$  can be determined. One gets

$$\begin{aligned} \pi^0 D^- (\pi^- D^+) &: \Delta \approx -7^\circ, \eta \approx 0.62, \theta \approx 26^\circ \\ \pi^- K^+ \text{ or } \pi^+ K^0 &: \Delta \approx -9^\circ, \eta \approx 0.52, \theta \approx 29^\circ \\ \pi^+ \pi^- &: \Delta \approx -21^\circ, \eta \approx 0.48, \theta \approx 31^\circ \end{aligned} \quad (19)$$

Hence we get the following bounds

$$\begin{aligned} \pi^0 D^- (\pi^- D^+) &: 0 \leq \delta_{f, \bar{f}} - \Delta \leq 26^\circ \\ \pi^- K^+ \text{ or } \pi^+ K^0 &: 0 \leq \delta_{f, \bar{f}} - \Delta \leq 29^\circ \\ \pi^+ \pi^- &: 0 \leq \delta_{f, \bar{f}} - \Delta \leq 31^\circ \end{aligned} \quad (20)$$

Further we note that for these decays,  $b$ -quark is converted into  $c$  or  $u$  quark :  $b \rightarrow c(u) + \bar{u} + d(s)$ . In particular for the tree graph, the configuration is such that  $\bar{u}$  and  $d(s)$  essentially go together into a color singlet state with the third quark  $c(u)$  recoiling; there is a significant probability that the system will hadronize as a two body final state.

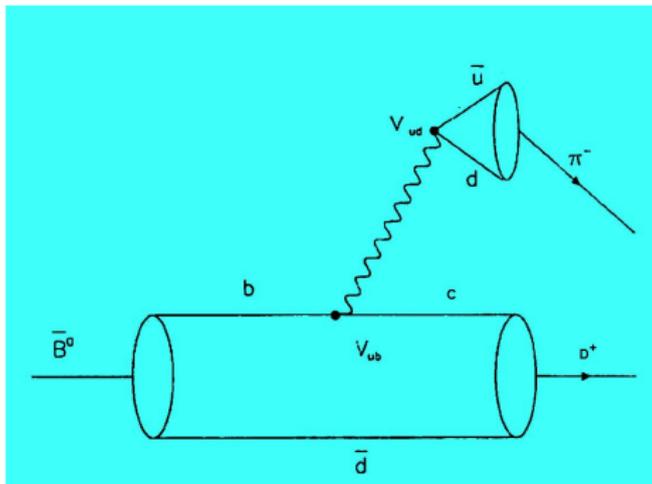


Figure: Tree diagram

This physical picture has been put on the strong theoretical basis, where in these references the QCD factorization have been proved. For the tree amplitude, factorization implies  $\delta_f^T = 0$ . We, therefore take the point of view that effective final state phase shift is given by  $\delta_f - \Delta$ . We take the lower bound for the tree amplitude so that final state effective phase shift  $\delta_f^T = 0$ . Thus for  $\pi^+ D^- (\pi^- D^+)$ ,  $\delta_f^T = \delta_f'^T = 0$ .

The decay  $B^0 \rightarrow \pi^- K^+$  is described by two amplitudes,

$$A(B^0 \rightarrow \pi^- K^+) = - [P + e^{i\gamma} T] = |P| \left[ 1 - r e^{i(\gamma + \delta_{+-})} \right] \quad (21)$$

where

$$P = - |P| e^{-i\delta_P}, \quad T = |T| e^{i\delta_T}, \quad \delta_{+-} = \delta_P, \quad r = \frac{|T|}{|P|}$$

For the direct  $CP$  asymmetries, the relevant phase is  $\delta_{+-}$ . For the penguin amplitude, we assume that the effective final state phase  $\delta_P$  has the value near the upper bound.

Thus we have  $\delta_{+-} \approx 29^\circ$ ,

Now

$$\begin{aligned} A_{CP}(B^0 \rightarrow \pi^- K^+) &= - \frac{2r \sin \gamma \sin \delta_{+-}}{R} \\ R &= 1 - 2r \cos \gamma \cos \delta_{+-} + r_{+-}^2 \end{aligned} \quad (22)$$

Neglecting the terms of order  $r^2$ , we have

$$\tan \gamma \tan \delta_{+-} = \frac{-A_{CP}(B^0 \rightarrow \pi^- K^+)}{1 - R} \quad (23)$$

Now the experimental values of  $A_{CP}$  and  $R$  are

$$A_{CP}(B^0 \rightarrow \pi^- K^+) = -0.101 \pm 0.015 (-0.097 \pm 0.012)$$

$$R = 0.899 \pm 0.048$$

where the numerical values in the bracket are the latest experimental values. With  $\delta_{+-} \approx 29^\circ$ , we get from Eq.(23),  $\gamma = (60 \pm 3)^\circ$ . However for  $\delta_{+-} \approx 20^\circ$  which one gets from Eq.(15) for  $\rho^2 = 0.65$ ,  $\gamma = (69 \pm 3)^\circ$ .

We conclude: The phase shift  $\delta_{+-} \approx (20 - 29)^\circ$  for  $\pi^- K^+$  is compatible with experimental value of the direct  $CP$ - asymmetry for  $\pi^- K^+$  decay mode. For  $\pi^+ \pi^-$ ,  $\delta_{+-} \sim 31^\circ$  is compatible with the experimental value  $(33 \pm 7_{-10}^{+8})^\circ$ . Finally we note that the actual value of the effective phase shift  $(\delta_f - \Delta)$  depends on one free parameter  $\rho$ , factorization implies  $\delta_f^T = 0$  i.e.  $\delta_f - \Delta = 0$  for the tree amplitude; for the penguin amplitude,  $\delta_f^P$  depends on  $\rho$ . However, from the experimental values of the direct  $CP$ -violation for  $\pi^- K^+$ ,  $\pi^- \pi^+$ , it is near the upper bound.

Finally we note that  $\pi^+ D^- (\pi^- D^+)$ ,  $\pi^- K^+$ ,  $\pi^- \pi^+$  decays are  $s$ -wave decay whereas  $B^0 \rightarrow \rho^+ \pi^- (\rho^- \pi^+)$  decays are  $p$ -wave decays. For  $p$ -wave, the decay amplitude

$$\begin{aligned}
 f &= \frac{1}{16\pi s} \int_{-s}^0 M(s, t) \left(1 + \frac{2t}{s}\right) dt \\
 &\simeq \frac{1}{16\pi s} \int_{-s}^0 M(s, t) dt
 \end{aligned}$$

Now for the  $B \rightarrow \rho\pi$  decay, only longitudinal polarization of  $\rho$  is effectively involved. Since the longitudinal  $\rho$ -meson emulates a pseudoscalar meson and if we assume same couplings as for pions, we conclude that the final state phase for  $\rho\pi$  should be of the order  $30^\circ$ ; in any case should not be greater than  $30^\circ$ . It is compatible with some values in Table-1 obtained from the  $B \rightarrow \rho\pi$  decay data.

# CP Asymmetries and Strong Phases

In this section, we discuss the experimental tests to verify the equality (implied by C-invariance of S-matrix) of phase shifts  $\delta_f$  and  $\delta_{\bar{f}}$  for the weak decays of B mesons mentioned in previous slides.

The effective Lagrangians  $\mathcal{L}_W$  and  $\mathcal{L}'_W$  are given by ( $q = d, s$ )

$$\mathcal{L}_W = V_{cb}V_{uq}^*[\bar{q}\gamma^\mu(1-\gamma^5)u][\bar{c}\gamma_\mu(1-\gamma_5)b]$$
$$\mathcal{L}'_W = V_{ub}V_{cq}^*[\bar{q}\gamma^\mu(1-\gamma^5)c][\bar{u}\gamma_\mu(1-\gamma_5)b] \quad (24)$$

(25)

$$A_f = \langle D^- \pi^+ | \mathcal{L}_W | B^0 \rangle = F_f$$
$$A_{\bar{f}} = \langle D^+ \pi^- | \mathcal{L}'_W | B^0 \rangle = e^{i\gamma} F'_{\bar{f}}$$
$$A_{f_s} = \langle K^+ D_s^- | \mathcal{L}_W | B_s^0 \rangle = F_{f_s}$$
$$A_{\bar{f}_s} = \langle K^- D_s^+ | \mathcal{L}'_W | B_s^0 \rangle = e^{i\gamma} F'_{\bar{f}_s} \quad (26)$$

Thus, for this case the CP-asymmetries are given:

$$\mathcal{A}(t) = -\frac{2r_D}{1+r_D^2} \sin \Delta m_{BT} \sin(2\beta + \gamma) \cos(\delta_f - \delta'_f)$$

$$\mathcal{F}(t) = \frac{1-r_D^2}{1+r_D^2} \cos \Delta m_{BT} - \frac{2r_D}{1+r_D^2} \sin \Delta m_{BT} \cos(2\beta + \gamma) \sin(\delta_f - \delta'_f) \quad (27)$$

$$\mathcal{A} = \frac{-2r_D}{1+r_D^2} \sin(2\beta + \gamma) \frac{(\Delta m_B/\Gamma)}{1 + (\Delta m_B/\Gamma)^2} \cos(\delta_f - \delta'_f) \quad (28)$$

where

$$r_D = \lambda^2 R_b \frac{|F'_f|}{|F_f|} \quad (29)$$

For the decays,

$$\bar{B}_s^0 (B_s^0) \rightarrow D_s^+ K^- (D_s^- K^+)$$

$$\bar{B}_s^0 (B_s^0) \rightarrow D_s^- K^+ (D_s^+ K^-)$$

we get,

$$\begin{aligned} \mathcal{A}_s(t) &= -\frac{2r_{D_s}}{1+r_{D_s}^2} \sin \Delta m_{B_s} t \sin(2\beta_s + \gamma) \cos(\delta_{f_s} - \delta'_{f_s}) \\ \mathcal{F}_s(t) &= \frac{1-r_{D_s}^2}{1+r_{D_s}^2} \cos \Delta m_{B_s} t - \frac{2r_{D_s}}{1+r_{D_s}^2} \sin \Delta m_{B_s} t \cos(2\beta_s + \gamma) \sin(\delta_{f_s} - \delta'_{f_s}) \end{aligned} \quad (30)$$

where

$$r_{D_s} = R_b \frac{|F'_{f_s}|}{|F_{f_s}|} \quad (31)$$

The experimental results for the B decays are as follows [?]

	$D^- \pi^+$	$D^{*-} \pi^+$	$D^- \rho^+$	
$\frac{S_- + S_+}{2}$ :	$-0.046 \pm 0.023$	$-0.037 \pm 0.012$	$-0.024 \pm 0.031 \pm 0.009$	(32)
$\frac{S_- - S_+}{2}$ :	$-0.022 \pm 0.021$	$-0.006 \pm 0.016$	$-0.098 \pm 0.055 \pm 0.018$	

where

$$\begin{aligned}\frac{S_- + S_+}{2} &\equiv -\frac{2r_D}{1+r_D^2} \sin(2\beta + \gamma) \cos(\delta_f - \delta'_f) \\ \frac{S_- - S_+}{2} &\equiv -\frac{2r_D}{1+r_D^2} \cos(2\beta + \gamma) \sin(\delta_f - \delta'_f)\end{aligned}\quad (33)$$

For  $B_s^0 \rightarrow D_s^{*-} K^+, D_s^- K^+, D_s^- K^{*+}$ , replace  $r_D \rightarrow r_s, \beta \rightarrow \beta_s, \delta_f \rightarrow \delta_{f_s}, \delta'_f \rightarrow \delta'_{f_s}$  in Eq. (33).

Since for  $B_s^0$ , in the standard model, with three generations, gives  $\beta_s = 0$ , so we have for the CP-asymmetries  $\sin \gamma$  or  $\cos \gamma$  instead of  $\sin(2\beta + \gamma), \cos(2\beta + \gamma)$ . Hence  $B_s^0$ -decays are more suitable for testing the equality of phase shifts  $\delta_{f_s}$  and  $\delta'_{f_s}$  as for this case neither  $r_s$  nor  $\cos \gamma$  is suppressed as compared to the corresponding quantities for  $B^0$ . To conclude, for  $B_q^0$  decays, the equality of phases  $\delta_f$  and  $\delta'_f$  for  $B_d^0$  gives

$$\begin{aligned}-\frac{S_- + S_+}{2} &= 2r_D \sin(2\beta + \gamma) \\ -\frac{S_- - S_+}{2} &= 0\end{aligned}\quad (34)$$

whereas for  $B_s^0$  decays, we get

$$\begin{aligned} -\frac{S_- + S_+}{2} &= \frac{2r_{D_s}}{1 + r_{D_s}^2} \sin(2\beta_s + \gamma) \\ -\frac{S_- - S_+}{2} &= 0 \end{aligned} \quad (35)$$

To determine the parameter  $r_D$  or  $r_{D_s}$ , we assume factorization for the tree amplitude. Factorization gives for the decays  $\bar{B}^0 \rightarrow D^+ \pi^-, D^{*+} \pi^-, D^+ \rho^-, D^+ a_1^-$ :

$$\begin{aligned} |\bar{F}_f| = |\bar{T}_f| &= G [f_\pi(m_B^2 - m_D^2) f_0^{B-D}(m_\pi^2), 2f_\pi m_B |\vec{p}| A_0^{B-D*}(m_\pi^2), \\ & 2f_\rho m_B |\vec{p}| f_+^{B-D}(m_\rho^2), 2f_{a_1} m_B |\vec{p}| f_+^{B-D}(a_1^2)] \end{aligned} \quad (36)$$

$$\begin{aligned} |\bar{F}'_f| = |\bar{T}'_f| &= G' [f_D(m_B^2 - m_\pi^2) f_0^{B-\pi}(m_D^2), 2f_{D^*} m_B |\vec{p}| f^{B-\pi}(m_{D^*}^2), \\ & 2f_D m_B |\vec{p}| A_0^{B-\rho}(m_D^2), 2f_D m_B |\vec{p}| A_0^{B-a_1}(m_B^2)] \end{aligned} \quad (37)$$

$$G = \frac{G_F}{\sqrt{2}} |V_{ud}| |V_{cb}| a_1, \quad G' = \frac{G_F}{\sqrt{2}} |V_{cd}| |V_{ub}| \quad (38)$$

The decay widths for the above channels are given in the table 1

Decay	Decay Width ( $10^{-9} \text{ MeV} \times  V_{cb} ^2$ )	Form Factor	Form Factors $h(v)$
$\bar{B}^0 \rightarrow D^+ \pi^-$	$(2.281) f_0^{B-D}(m_\pi^2) ^2$	$0.58 \pm 0.05$	$0.51 \pm 0.03$
$\bar{B}^0 \rightarrow D^{*+} \pi^-$	$(2.129) A_0^{B-D^*}(m_\pi^2) ^2$	$0.61 \pm 0.04$	$0.54 \pm 0.03$
$\bar{B}^0 \rightarrow D^+ \rho^-$	$(5.276) f_+^{B-D}(m_\rho^2) ^2$	$0.65 \pm 0.11$	$0.57 \pm 0.10$
$\bar{B}^0 \rightarrow D^+ a_1^-$	$(5.414) f_+^{B-D}(m_{a_1}^2) ^2$	$0.57 \pm 0.31$	$0.50 \pm 0.27$

Table: Form Factors

where we have used

$$a_1^2 |V_{ud}|^2 \approx 1, \quad f_\pi = 131 \text{ MeV}, \quad f_\rho = 209 \text{ MeV}, \quad f_{a_1} = 229 \text{ MeV}$$

Using the experimental branching ratios and

$$|V_{cb}| = (38.3 \pm 1.3) \times 10^{-3} \quad (39)$$

we obtain the corresponding form factors given in Table 1.

In terms of variables:

$$\omega = v \cdot v', \quad v^2 = v'^2 = 1, \quad t = q^2 = m_B^2 + m_{D^{(*)}}^2 - 2m_B m_{D^{(*)}} \omega \quad (40)$$

the form factors can be put in the following form

$$\begin{aligned}
 f_+^{B-D}(t) &= \frac{m_B + m_D}{2\sqrt{m_B m_D}} h_+(\omega), & f_0^{B-D}(t) &= \frac{\sqrt{m_B m_D}}{m_B + m_D} (1 + \omega) h_0(\omega) \\
 A_2^{B-D^*}(t) &= \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} (1 + \omega) h_{A_2}(\omega), & A_0^{B-D^*}(t) &= \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} h_{A_0}(\omega) \\
 A_1^{B-D^*}(t) &= \frac{\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}} (1 + \omega) h_{A_1}(\omega)
 \end{aligned} \tag{41}$$

Heavy Quark Effective Theory (HQET) gives:

$$h_+(\omega) = h_0(\omega) = h_{A_0}(\omega) = h_{A_1}(\omega) = h_{A_2}(\omega) = \zeta(\omega)$$

where  $\zeta(\omega)$  is the form factor, with normalization  $\zeta(1) = 1$ . For

$$\begin{aligned}
 t &= m_\pi^2, m_\rho^2, m_{a_1}^2 \\
 \omega^{(*)} &= 1.589(1.504), 1.559, 1.508
 \end{aligned} \tag{42}$$

The value for  $h_{A_1}(\omega_{max}^*)$  obtained from the analysis of  $B^0 \rightarrow D^{(*)} l^- \nu_e$

$$|h_{A_1}(\omega_{max}^*)| = 0.52 \pm 0.03 \tag{43}$$

Since  $\omega_{max}^* = 1.504$ , the value for  $|h_{A_0}(\omega_{max}^*)|$  obtained in Table 1 is in remarkable agreement with the value given in Eq. (43) showing that factorization assumption for  $B^0 \rightarrow \pi D^{(*)}$  decays is experimentally on solid footing and is in agreement with HQET.

From Eqs. (36) and (37), we obtain

$$r_D = \lambda^2 R_b \frac{|\bar{T}'_f|}{|\bar{T}_{\bar{f}}|} = \lambda^2 R_b \left[ \frac{f_D(m_B^2 - m_\pi^2) f_0^{B-\pi}(m_D^2)}{f_\pi(m_B^2 - m_D^2) f_0^{B-D}(m_\pi^2)}, \quad \frac{f_{D^*} f_+^{B-\pi}(m_{D^*}^2)}{f_\pi A_0^{B-D}(m_\pi^2)}, \quad \frac{f_D A_0^{B-\rho}(m_D^2)}{f_\rho f_+^{B-D}(m_\rho^2)} \right] \quad (44)$$

where

$$\frac{|V_{ub}| |V_{cd}|}{|V_{cb}| |V_{ud}|} = \lambda^2 R_b \approx (0.227)^2 (0.40) \approx 0.021 \quad (45)$$

From (44), we get

$$r_{D^{(*)}} = [0.018 \pm 0.002, \quad 0.017 \pm 0.003, \quad 0.012 \pm 0.002] \quad (46)$$

The above value for  $r_D^*$  gives

$$-\left(\frac{S_+ + S_-}{2}\right)_{D^*\pi} = 2(0.017 \pm 0.003) \sin(2\beta + \gamma) \quad (47)$$

The experimental value of the CP asymmetry for  $B^0 \rightarrow D^*\pi$  decay has the least error. Hence we obtain the following bounds

$$\sin(2\beta + \gamma) > 0.69 \quad (48)$$

$$44^\circ \leq (2\beta + \gamma) \leq 90^\circ \quad (49)$$

$$\text{or } 90^\circ \leq (2\beta + \gamma) \leq 136^\circ \quad (50)$$

Selecting the second solution, and using  $2\beta \approx 43^\circ$ , we get

$$\gamma = (70 \pm 23)^\circ \quad (51)$$

For  $\bar{B}_s^0 \rightarrow D_s^{*+} K^-$

$$r_{D_s} = R_b \left[ \frac{f_{D_s^*} f_+^{B_s - K} (m_{D_s^*}^2)}{f_K A_0^{B_s - D_s^*} (m_K^2)} \right] \quad (52)$$

Hence we get

$$\begin{aligned} -\left(\frac{S_+ + S_-}{2}\right)_{D_s^* K} &= (0.41 \pm 0.08) \sin(2\beta_s + \gamma) \\ &= (0.41 \pm 0.08) \sin \gamma \end{aligned} \tag{53}$$

## CP Asymmetries for $A_f \neq A_{\bar{f}}$

$$B^0(\bar{B}^0) \rightarrow \rho^- \pi^+, \rho^+ \pi^-$$

We now discuss the decays listed in case-II where  $A_f \neq A_{\bar{f}}$ .

$$\begin{aligned} \frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} &= C_f \cos \Delta mt + S_f \sin \Delta mt \\ &= (C - \Delta C) \cos \Delta mt + (S - \Delta S) \sin \Delta mt \end{aligned} \quad (54)$$

$$\begin{aligned} \frac{\Gamma_{\bar{f}}(t) - \bar{\Gamma}_{\bar{f}}(t)}{\Gamma_{\bar{f}}(t) + \bar{\Gamma}_{\bar{f}}(t)} &= C_{\bar{f}} \cos \Delta mt + S_{\bar{f}} \sin \Delta mt \\ &= (C + \Delta C) \cos \Delta mt + (S + \Delta S) \sin \Delta mt \end{aligned} \quad (55)$$

where  $C_{\bar{f},f}$  are direct  $CP$  asymmetries  $S_{f,\bar{f}}$  are mixing induces  $CP'$ 's.

The following relations are also useful:

$$\frac{R_{\bar{f},f}}{R_f + R_{\bar{f}}} = \frac{1}{2} [(1 \pm \Delta C) \pm A_{CP}C] \quad (56)$$

$$\frac{R_{\bar{f}} - R_f}{R_f + R_{\bar{f}}} = [\Delta C + A_{CP}C] \quad (57)$$

(58)

$$\frac{R_{\bar{f}}A_{CP}^{\bar{f}} + R_f A_{CP}^f}{R_f + R_{\bar{f}}} = [C + A_{CP}\Delta C] \quad (59)$$

$$A_{CP}^{\bar{f}} = \frac{\bar{\Gamma}_f - \Gamma_{\bar{f}}}{\bar{\Gamma}_{\bar{f}} + \bar{\Gamma}_f}$$

$$A_{CP}^f = \frac{\bar{\Gamma}_{\bar{f}} - \Gamma_f}{\Gamma_f + \bar{\Gamma}_{\bar{f}}} \quad (60)$$

$$A_{CP} = \frac{(\bar{\Gamma}_{\bar{f}} + \bar{\Gamma}_f) - (\bar{\Gamma}_f + \Gamma_f)}{(\bar{\Gamma}_{\bar{f}} - \bar{\Gamma}_f) - (\bar{\Gamma}_f + \Gamma_f)} \quad (61)$$

$$= \frac{R_f A_{CP}^f - R_{\bar{f}} A_{CP}^{\bar{f}}}{\Gamma} \quad (62)$$

where

$$R_f = \frac{1}{2}(\Gamma_f + \bar{\Gamma}_{\bar{f}}), \quad R_{\bar{f}} = \frac{1}{2}(\Gamma_{\bar{f}} + \bar{\Gamma}_f)$$

$$\Gamma = R_f + R_{\bar{f}} \quad (63)$$

For  $B^0 \rightarrow \rho^- \pi^+$ ,  $B^0 \rightarrow \rho^+ \pi^-$ , we have

$$A_f = |T_f| e^{-i\gamma} e^{i\delta_f^T} [1 - r_f e^{i(\alpha + \delta_f)}]$$

$$A_{\bar{f}} = |T_{\bar{f}}| e^{-i\gamma} e^{i\delta_{\bar{f}}^T} [1 - r_{\bar{f}} e^{i(\alpha + \delta_{\bar{f}})}] \quad (64)$$

where

$$r_{f\bar{f}} = \frac{|V_{tb}||V_{td}|}{|V_{ub}||V_{ud}|} \frac{|P_{f,\bar{f}}|}{|T_{f,\bar{f}}|} = \frac{R_t}{R_b} \frac{|P_{f,\bar{f}}|}{|T_{f,\bar{f}}|} \quad (65)$$

The experimental results for these decays are:

$$\Gamma = R_f + R_{\bar{f}} = (22.8 \pm 2.5) \times 10^{-6} \quad (66)$$

$$A_{CP}^f = -0.16 \pm 0.23, \quad A_{CP}^{\bar{f}} = 0.08 \pm 0.12 \quad (67)$$

$$C = 0.01 \pm 0.14, \quad \Delta C = 0.37 \pm 0.08 \quad (68)$$

$$S = 0.01 \pm 0.09, \quad \Delta S = -0.05 \pm 0.10 \quad (69)$$

With the above values, it is hard to draw any reliable conclusion. Neglecting the term  $A_{CP}C$  in Eqs. (56) and (57), we get

$$R_{\bar{f}f} = \frac{1}{2}\Gamma(1 \pm \Delta C) \quad (70)$$

$$R_{\bar{f}} - R_f = \Delta C \quad (71)$$

$$(72)$$

Using the above value for  $\Delta C$ , we obtain

$$R_{\bar{f}} = (15.6 \pm 1.7) \times 10^{-6}$$

$$R_f = (7.2 \pm 0.8) \times 10^{-6} \quad (73)$$

We analyze these decays by assuming factorization for the tree graphs. This assumption gives

$$T_{\bar{f}} = \bar{T}_f \sim 2m_B f_\rho |\vec{p}| f_+(m_\rho^2) \quad (74)$$

$$T_f = \bar{T}_{\bar{f}} \sim 2m_B f_\pi |\vec{p}| A_0(m_\pi^2) \quad (75)$$

Using  $f_+(m_\rho^2) \approx 0.26 \pm 0.04$  and  $A_0(m_\pi^2) \approx A_0(0) = 0.29 \pm 0.03$  and  $|V_{ub}| = (3.5 \pm 0.6) \times 10^{-3}$ , we get the following values for the tree amplitude contribution to the branching ratios

$$\Gamma_{\bar{f}}^{\text{tree}} = (15.6 \pm 1.1) \times 10^{-6} \equiv |T_{\bar{f}}|^2 \quad (76)$$

$$\Gamma_f^{\text{tree}} = (7.6 \pm 1.4) \times 10^{-6} \equiv |T_f|^2 \quad (77)$$

$$t = \frac{T_f}{T_{\bar{f}}} = \frac{f_\pi A_0(m_\pi^2)}{f_\rho f_+(m_\rho^2)} = 0.70 \pm 0.12 \quad (78)$$

$$B_{\bar{f}} = \frac{R_{\bar{f}}}{|T_{\bar{f}}|^2} = 1 - 2r_{\bar{f}} \cos \alpha \cos \delta_{\bar{f}} + r_{\bar{f}}^2 = 1.00 \pm 0.12 \quad (79)$$

$$B_f = \frac{R_f}{|T_f|^2} = 1 - 2r_f \cos \alpha \cos \delta_f + r_f^2 = 0.95 \pm 0.11 \quad (80)$$

In order to take into account the contribution of penguin diagram, we introduce the angles  $\alpha_{\text{eff}}^{f, \bar{f}}$ , defined as follows

$$\begin{aligned} e^{i\beta} A_{f, \bar{f}} &= |A_{f, \bar{f}}| e^{-i\alpha_{\text{eff}}^{f, \bar{f}}} \\ e^{-i\beta} \bar{A}_{\bar{f}, f} &= |\bar{A}_{\bar{f}, f}| e^{i\alpha_{\text{eff}}^{f, \bar{f}}} \end{aligned} \quad (81)$$

With this definition, we separate out tree and penguin contributions:

$$\begin{aligned}
 e^{i\beta} A_{f,\bar{f}} - e^{-i\beta} \bar{A}_{\bar{f},f} &= |A_{f,\bar{f}}| e^{-i\alpha^f \bar{f}} - |\bar{A}_{\bar{f},f}| e^{i\alpha^f \bar{f}} \\
 &= 2iT_{f,\bar{f}} \sin \alpha
 \end{aligned} \tag{82}$$

$$\begin{aligned}
 e^{i(\alpha+\beta)} A_{f,\bar{f}} - e^{-i(\alpha+\beta)} \bar{A}_{\bar{f},f} &= |A_{f,\bar{f}}| e^{-i(\alpha_{eff}^{f,\bar{f}} - \alpha)} - |\bar{A}_{\bar{f},f}| e^{-i(\alpha_{eff}^{f,\bar{f}} - \alpha)} \\
 &= (2iT_{f,\bar{f}} \sin \alpha) r_{f,\bar{f}} e^{i\delta_{f,\bar{f}}} \\
 &= 2iP_{f,\bar{f}} \sin \alpha
 \end{aligned} \tag{83}$$

Now factorization implies

$$\delta_f^T = 0 = \delta_{\bar{f}}^T \tag{84}$$

In the limit  $\delta_f^T \rightarrow 0$ , we get:

$$\cos 2\alpha_{eff}^{f,\bar{f}} = -1, \quad \alpha_{eff}^{f,\bar{f}} = 90^\circ \quad (85)$$

$$r_{f,\bar{f}} \cos \delta_{f,\bar{f}} = \cos \alpha \quad (86)$$

$$r_{f,\bar{f}} \sin \delta_{f,\bar{f}} = \frac{-A_{CP}^{f,\bar{f}} / \sin \alpha}{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}}} \quad (87)$$

$$r_{f,\bar{f}}^2 = \frac{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}} \cos 2\alpha}{1 + \sqrt{1 - A_{CP}^{f,\bar{f}2}}} \quad (88)$$

$$\approx \cos^2 \alpha + \frac{1}{4} A_{CP}^{f,\bar{f}2} \sin^2 \alpha \quad (89)$$

The solution of Eq. (86) is graphically shown in Fig. 1 for  $\alpha$  in the range  $80^\circ \leq \alpha < 103^\circ$  for  $r_{f,\bar{f}} = 0.10, 0.15, 0.20, 0.25, 0.30$ . From the figure, the final state phases  $\delta_{f,\bar{f}}$  for various values of  $r_{f,\bar{f}}$  can be read for each value of  $\alpha$  in the above range. Few examples are given in Table 2

$\alpha$	$r_f$	$\delta_f$	$A_{CP}^f \approx -2r_f \sin \delta_f \sin \alpha$
80°	0.20	29°	-0.19
	0.25	46°	-0.36
82°	0.15	22°	-0.11
	0.20	46°	-0.28
85°	0.10	29°	-0.10
	0.15	54°	-0.24
86°	0.10	46°	-0.14
	0.15	62°	-0.26
88°	0.10	70°	-0.19

Table:

These examples have been selected keeping in view that final state phases  $\delta_{f,\bar{f}}$  are not too large.

Neglecting terms of order  $r_{f,\bar{f}}^2$ , we have

$$A_{CP} \approx \frac{2 \sin \alpha (r_{\bar{f}} \sin \delta_{\bar{f}} - t^2 r_f \sin \delta_f)}{1 + t^2} = -\frac{A_{CP}^{\bar{f}} - t^2 A_{CP}^f}{1 + t^2} \quad (90)$$

$$C \approx -\frac{2t^2}{(1+t)^2} (A_{CP}^{\bar{f}} + A_{CP}^f) \quad (91)$$

$$\Delta C \approx \frac{1-t^2}{1+t^2} - \frac{4t^2 \cos \alpha}{(1+t^2)^2} (r_{\bar{f}} \cos \delta_{\bar{f}} - r_f \cos \delta_f) \quad (92)$$

Now the second term in Eq. (92) vanishes and using the value of  $t$ , we get

$$\Delta C \approx 0.34 \pm 0.06 \quad (93)$$

in agreement with the experimental value.

Finally the CP asymmetries in the limit  $\delta_{f,\bar{f}}^T \rightarrow 0$

$$\begin{aligned}
 S_{\bar{f}} = S + \Delta S &= \frac{2\text{Im}[e^{2i\phi_M} A_{\bar{f}^*} \bar{A}_{\bar{f}}]}{\Gamma(1 + A_{CP})} \\
 &= \sqrt{1 - C_{\bar{f}}^2} \sin(2\alpha_{eff}^{\bar{f}} + \delta) \\
 &= -\sqrt{1 - C_{\bar{f}}^2} \cos \delta
 \end{aligned} \tag{94}$$

$$\begin{aligned}
 S_f = S - \Delta S &= \frac{2\text{Im}[e^{2i\phi_M} A_f^* \bar{A}_f]}{\Gamma(1 - A_{CP})} \\
 &= \sqrt{1 - C_f^2} \sin(2\alpha_{eff}^f - \delta) \\
 &= \sqrt{1 - C_f^2} \cos \delta
 \end{aligned} \tag{95}$$

The phase  $\delta$  is defined as

$$\bar{A}_{\bar{f}} = \frac{|\bar{A}_{\bar{f}}|}{|A_f|} \bar{A}_f e^{i\delta} \tag{96}$$

# Conclusion I

- No evidence that space-time symmetries are violated by fundamental laws of nature. Both translational and rotational symmetries hold in nature.

Translational Symmetry  $\Rightarrow$  Space-time is homogenous

$\Rightarrow$  Energy Momentum Conservation

Rotational Symmetry  $\Rightarrow$  Space isotropic

$\Rightarrow$  Angular Momentum Conservation

- If we examine light emitted by a distant object billions of light years away, we find that atoms have been following the same laws as they are here and now. (Translation Symmetry)
- Discrete Symmetries are not universal; both C and P are violated in the weak interaction but respected by electromagnetic and strong interactions. There is no evidence for violation of time reversal invariance by any of the fundamental laws of nature.

## Conclusion II

- Basic weak interaction Lagrangian is CP conserving. CP violation in weak interactions is a consequence of mismatch between mass eigenstates and CP eigenstates and or mismatch between weak and mass eigenstates at quark level. There is no evidence of CP violation in Lepton sector.
- CP violation in weak decays is an example where basic laws are CP invariant but states at quark level violate CP.
- The fundamental interaction governing the atoms and molecules is the electromagnetic interaction which does not violate bilateral symmetry (left-right symmetry). In nature we find organic molecules in asymmetric form, i.e. left handed or right handed. This is another example where the basic laws governing these molecules are bilateric symmetric but states are not. (Asymmetric initial conditions?)
- **Baryon Asymmetry of the Universe: Baryogenesis**  
No evidence for existence of antibaryons in the universe.  
 $\eta = n_B/n_\gamma \sim 3 \times 10^{-10}$ . The universe started with a complete matter antimatter symmetry in big bang picture. In subsequent evolution of the universe, a net baryon number was generated. This is possible provided

## Conclusion III

- ① There exists a baryon number violating interaction.
  - ② There exist C and CP violation to introduce the asymmetry between particle and antiparticle processes.
  - ③ Departure from thermal equilibrium of X-particles which mediate the baryon number violating interactions.
- There seems to be no connection between CP violation required by baryogenesis and CP violation observed in weak decays.