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Adiabatic model for dust atoms and molecules

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Introduction

- Although the 1% part of universe has 99% plasma, there are few astrophysical problems where plasma physics solutions have been suggested.

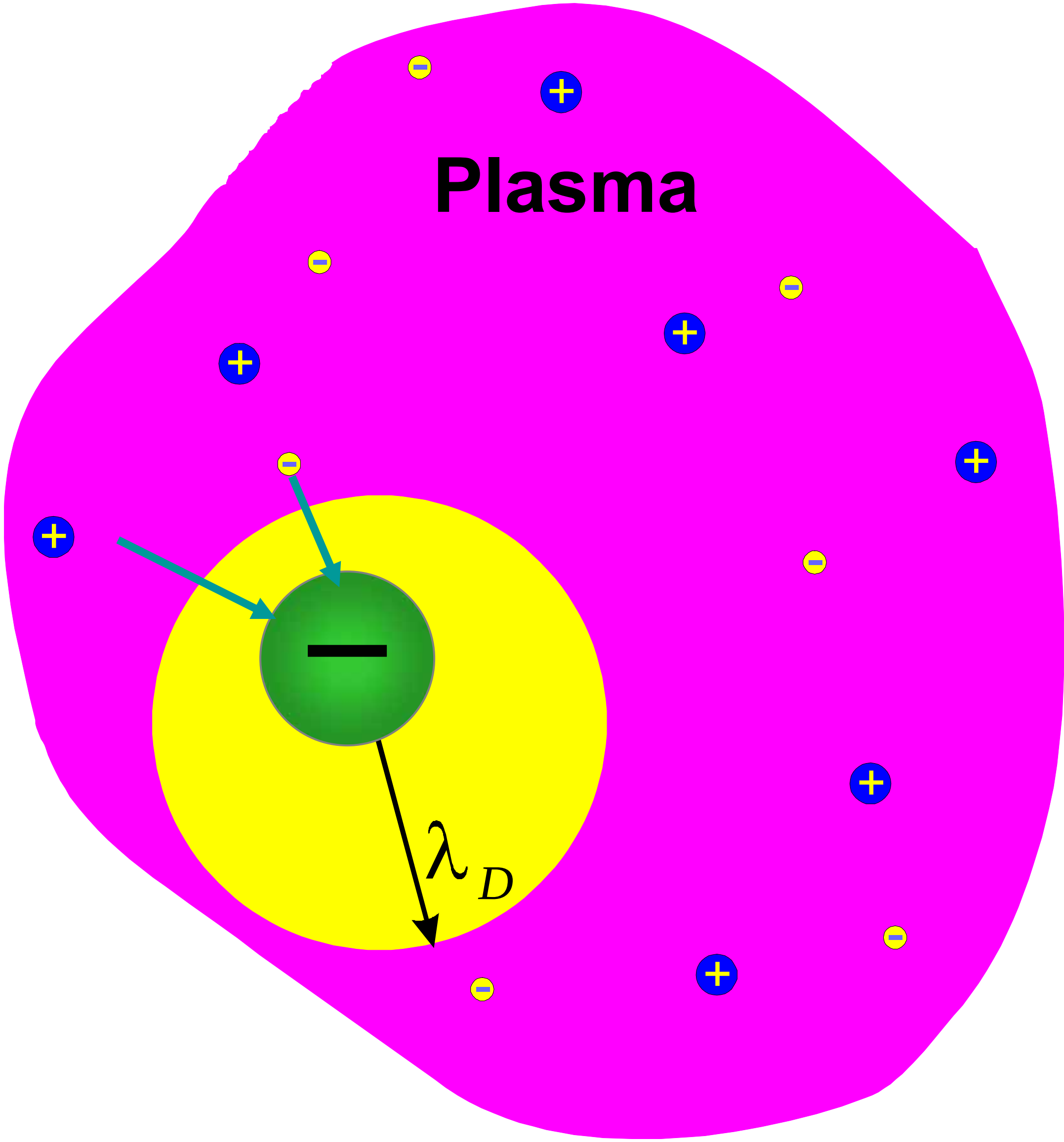
- Astrophysical plasma coexists with dust particles in many situations.

- These particles are charged either negatively or positively depending on their surrounding plasma environments.

Dusty plasmas

- ✓ electrons + ions
  + small particle
  of solid matter
- ✓ A fully or partially
  ionized plasma.
- ✓ Highly massive
  ($m_d \sim 10^6 - 10^{18}\, m_p$)
  Highly charged
  ($q \sim 10^3 - 10^4\, e$)
- ✓ **Variable Charge**

$$\frac{\partial q}{\partial t} = I(\mathbf{r}, q, t)$$



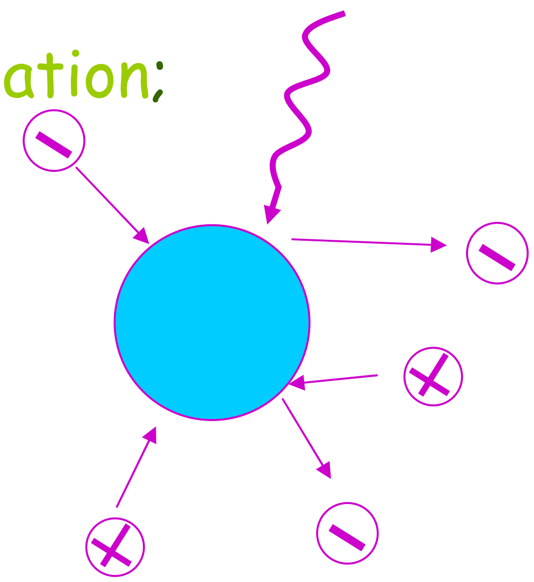
The total charging current

$$I(\mathrm{r},q,t) = I_{ext} + \sum_{\beta=e,i} I_{\beta}(\mathbf{r},q,t)$$

where I_{β} is the electronic or ionic current and

I_{ext} are external currents due to:

- Photoemission by incidence of UV radiation;
- secondary electron emission;
- thermionic emission etc.



Some novel aspects of dust in plasmas (dust atoms & molecules)

We discuss Nonlinear Screening of dust grains in a homogenous fully ionized electron-ion plasma under the following headings:

- Adiabatic Processes
- Thomas- Fermi Model for Dust Atom
- Motion of Particle in a Central Field
- Dust Molecule

The Model

we assume that the electrons and ions are inertialess

$$\nabla e|\varphi| + \frac{1}{n_e}\nabla P_e \quad = 0$$

$$\nabla Z_i e|\varphi| - \frac{1}{n_i}\nabla P_i = 0$$

Adiabatic Process

$$\boxed{PV^{\gamma} = const}$$

$$\frac{T^{3/2}}{n} = const\,, \qquad \frac{P}{n^{5/3}} = const$$

P_e and P_i can be expressed in term of density as

$$\mathrm{P_e} = n_{oe} T_{oe} \left(\frac{n_e}{n_{oe}} \right)^{5/3} ; \qquad \mathrm{P_i} = n_{oi} T_{oi} \left(\frac{n_i}{n_{oi}} \right)^{5/3}$$

Where $n_{o\alpha}$ and $T_{o\alpha}$ are the mean density and temperature of the species (e,i)

we obtain the **densities** of electrons and ions

$$\frac{n_e}{n_{oe}} = \left(1 - \frac{2}{5} \frac{e\,|\,\varphi\,|}{T_{oe}} \right)^{3/2} ; \qquad \frac{n_i}{n_{oi}} = \left(1 + \frac{2}{5} Z_i \frac{e\,|\,\varphi\,|}{T_{oi}} \right)^{3/2}$$

To calculate electrostatic potential field, we use the Poisson equation

$$\nabla^2 |\varphi| = 4\pi e\,(Z_i n_i - n_e)$$

- In a region far from the dust grain **$e|\varphi| < T_e, T_i$**

so that densities in the approximate form become:

$$n_e = n_{0e} - \frac{3}{5}\frac{e|\varphi|}{T_{0e}} \qquad n_i = n_{oi} + \frac{3}{5}\frac{e|\varphi|}{T_{oi}}$$

Debye Potential

$$|\varphi| = \frac{Z_D e}{r}\exp\left(-\frac{r}{\lambda_D}\right)$$

$\lambda_D > \lambda^B_D$ as

$$\lambda_D = \left(\sqrt{\frac{5}{3}}\right)\lambda_D^B$$

Adiabatic model and charging process

$$\frac{n_e}{n_{oe}} = \left(1 - \frac{2}{5}\frac{e\,|\,\varphi\,|}{T_{oe}}\right)^{3/2}$$

- In the vicinity of grain surface, the electrons having less thermal velocities can not penetrate into the potential barrier of the dust grain. the maximum potential field to be

$$|\,\varphi\,|_{Max} = \frac{5}{2}\frac{T_{oe}(ev)}{e^2}$$

on the other hand, the potential field becomes maximum only on the surface of dust grain, i.e.,

$$|\,\varphi\,|_{Max} = \frac{Z_D}{r_D}e$$

Thus we obtain :

$$Z_D = 2.5 \frac{T_{oe}}{e^2} r_D$$

Validity

An Important relation between the charge number and the temperature, for a given radius of the dust grain.

Using different values of the temperature of electrons and the radius of the grains, we calculated the magnitude of the charge Z_D from above relation for various plasma environments and found it in good agreement with the values cited in the Mendis table

How a large number of ions will circumnavigate the dust grain?

If $T_{oe} \sim T_{oi} = T_o$

$$n_i = n_{oi}(1 + Z_i)^{3/2}$$

If $T_{oe} \neq T_{oi}$ and $T_{oe} > T_{oi}$

$$n_i = n_{oi}\left(1 + Z_i \frac{T_{oe}}{T_{oi}}\right)^{3/2} \approx n_{oi}\left(Z_i \frac{T_{oe}}{T_{oi}}\right)^{3/2}$$

As $T_i < e|\varphi|$, the ions will mostly remain close to the surface

In a region close to the dust grain surface, $e|\varphi|>T_i$ leads to nonlinear screening associated with the trapped ions population which can be formed around the dust grain.

In the electron-proton plasma, in the vicinity of the grains, we can write the Poisson equation for vanishingly small n_e

$$\nabla^2\Phi = (1+\Phi)^{3/2}$$

$$\frac{2Z_i e}{5T_i}|\varphi|$$

Introducing a new function $1+\Phi = F$ we obtain the **Thomas-Fermi equation** i.e.,

$$\nabla^2 F = F^{3/2}$$

The function Φ itself will satisfy this equation when the ions temperature is less than that of electrons so that we can neglect unity in comparison with Φ.

A simple picture of the motion of protons around the dust grain

Using $T_e \sim T_i$ and taking $\Phi \approx 1$ on the r.h.s. which means that the electrons are pushed out of the region and only the protons reside close to the dust grain.

Thus

$$\nabla^2 \Phi = 2^{3/2}$$

Which has solution

$$\Phi = \frac{\sqrt{2}}{3} \frac{r^2}{\rho^2}$$

$$\frac{5}{2} \frac{T}{4\pi n_o e^2}$$

Consequently the protons will have Potential energy

$$U_i = \frac{1}{2} m_i \omega_{pi}^2 \, r^2$$

$$\boxed{\omega_{pi}^2 = 2^{3/2} \frac{4\pi \, n_{oi} e^2}{3m_i}}$$

Ion Langmuir frequency

Protons execute oscillations like <u>Harmonic Oscillators</u>

The standard 3-D oscillator solution gives

Energy levels

$$E_n = (n + \frac{3}{2})\eta\omega$$

Where n = 0,1,2...

Then the wave function of the normal ground state

$$\Psi_o(r) \sim e^{-r^2/2\rho_o^2}$$

Where ρ_o is the oscillation length of the proton

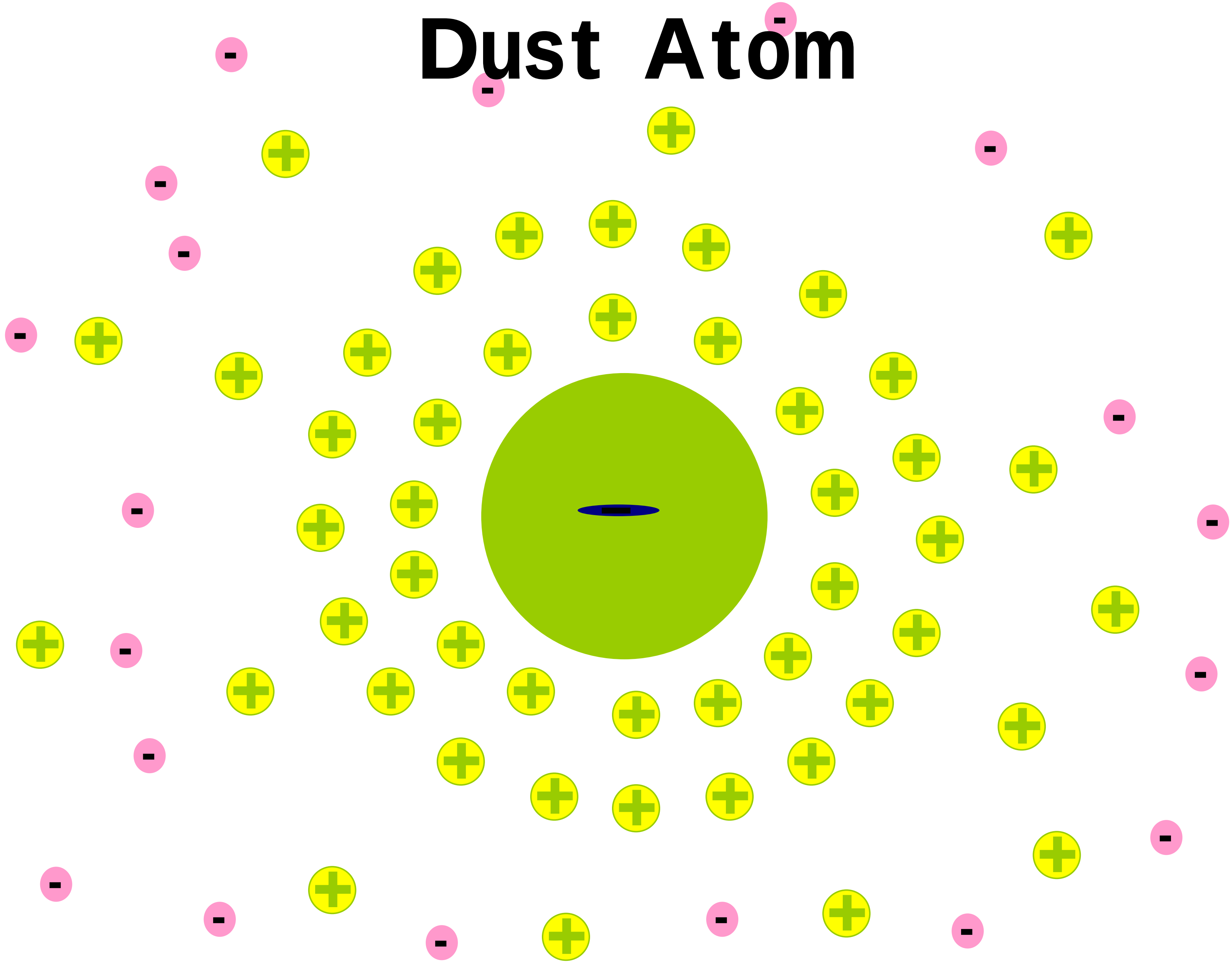
$$\rho_o \sim \sqrt{\frac{2\eta}{m_i\omega_o}}$$

~10^{-3} cm

Conclusion

protons are oscillating in the vicinity of dust grain at a distance $(10^{-8} - 10^{-5} m)$ larger than the dust grain size so the existence of such gigantic atoms is indeed a possibility

Dust Atom



Size of the atom

Effective potential energy

$$U_{eff}(r) = U(r) + \frac{L^2}{2m_e r^2}$$

For the electron density

$$\frac{n_e}{n_{oe}} = \left(1 - \frac{2}{5}\frac{Z_D e^2}{T_e r} - \frac{L^2}{5m_e T_e r^2}\right)^{3/2}$$

$$L = mV_l r_l = \eta l$$ where $l = 1, 2, 3\ldots$

$$\frac{n_e}{n_{oe}} = \sum \left(1 - \frac{2}{5}\frac{Z_D e^2}{T_e r_l} - \frac{l^2 \eta^2}{5m_e T_e r_l^2}\right)^{3/2}$$

$$r_l = r_D + \frac{1}{2} l^2 \frac{\eta^2}{Z_D m_e e^2}$$

If all the energy levels are filled with protons, the number of orbits will be the same as the charge number i.e.,

$$l = Z_D$$

$$r_{last} = r_{Z_D} - r_D = \boxed{\frac{Z_D \eta^2}{2 m_e e^2}}$$

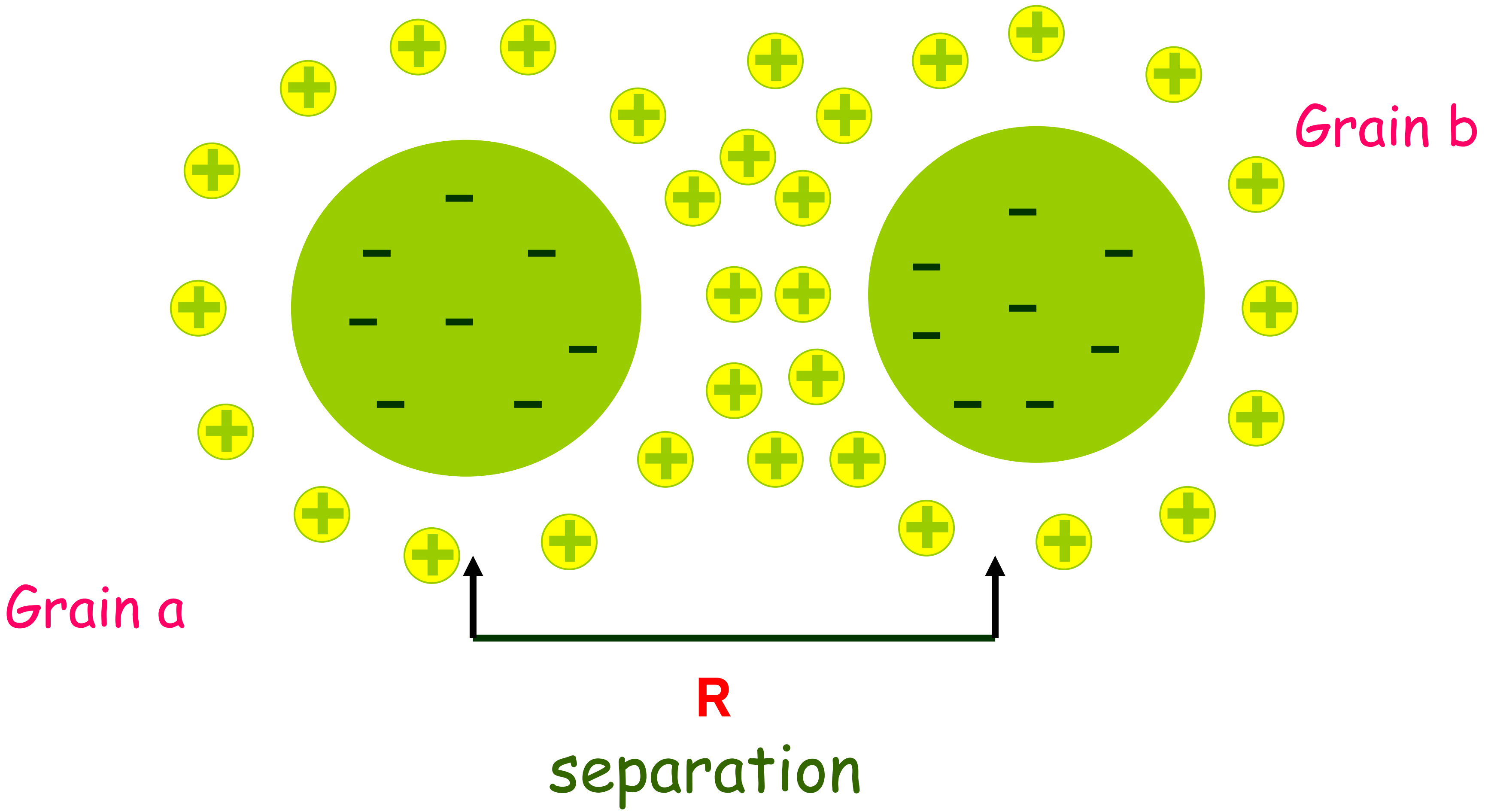
Evidently $r_{last} << \lambda_D$

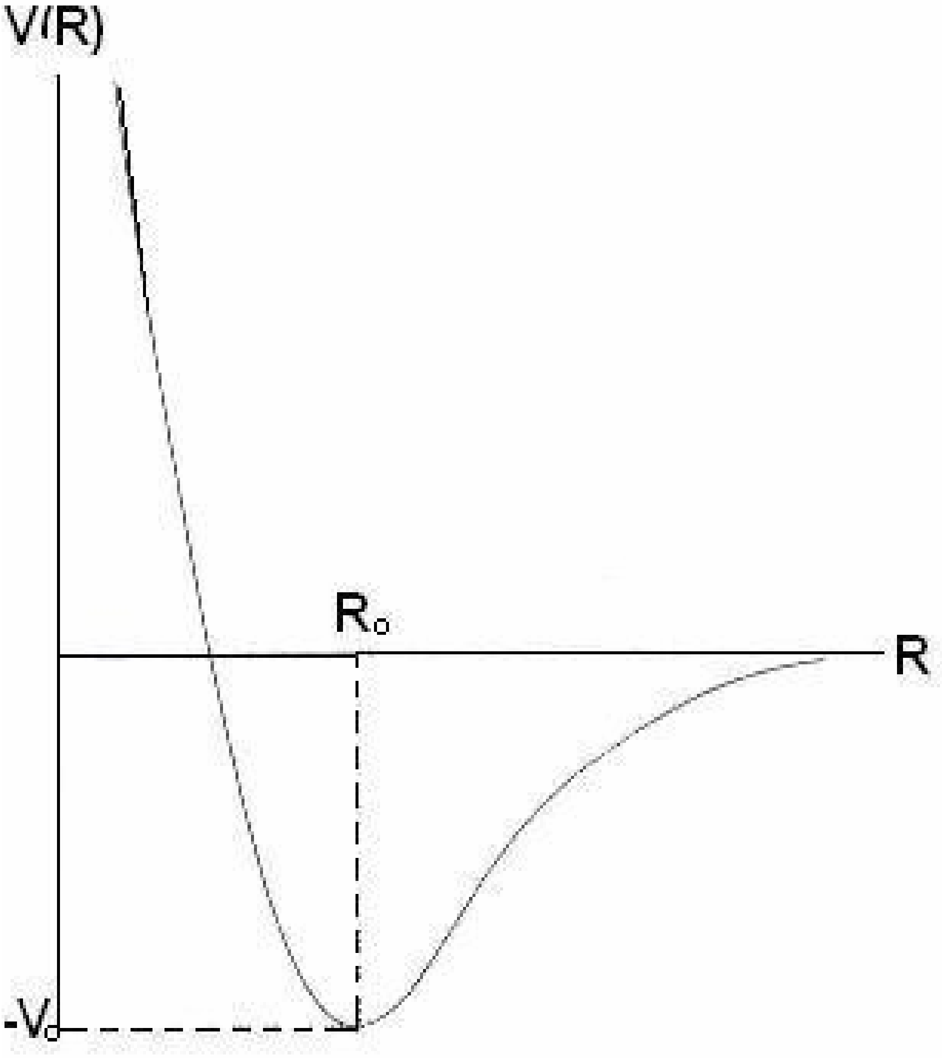
Dust Molecule

There are many physical systems where the harmonic oscillator solution is applicable. One such system is a diatomic molecule in which the two atoms vibrate approximately harmonically along the line joining the two atoms.

In quantum theory, molecular structure is described by the well known
Born-Oppenheimer approximation

Two dust atoms in a plasma





Potential energy V(R) having general features: the dip in V(R) provides an attractive well that may be able to support bound states, a short-range repulsion, asymptotically becomes zero on the large R.

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Plot of the potential energy V(R) an distance R between two dust atoms of a molecule.

To investigate this process, we expand V(R) about the equilibrium position R_o

$$V(R) = -V_o(R_o) + \frac{(R-R_o)^2}{2}\left(\frac{\partial^2 V(R)}{\partial R^2}\right)_{R=R_o} + \ldots$$

Where 1st term is the attractive portion which has minimum value $-\mathbf{V_o}$ at the average separation R_o. 2nd term gives angular frequency of two dust grains.

$$\omega = \left[\frac{1}{\mu}\left(\frac{\partial^2 V(R)}{\partial R^2}\right)_{R=R_o}\right]^{1/2}$$

Protonic Energy

$$E_{pro} = V(R_o) \approx \frac{(\Delta P)^2}{2m_i}$$

$$\Delta P \sim \frac{\eta}{R_o}$$

momentum uncertainty

$$E_{pro} \sim \frac{\eta^2}{2 m_i R_o^2}$$

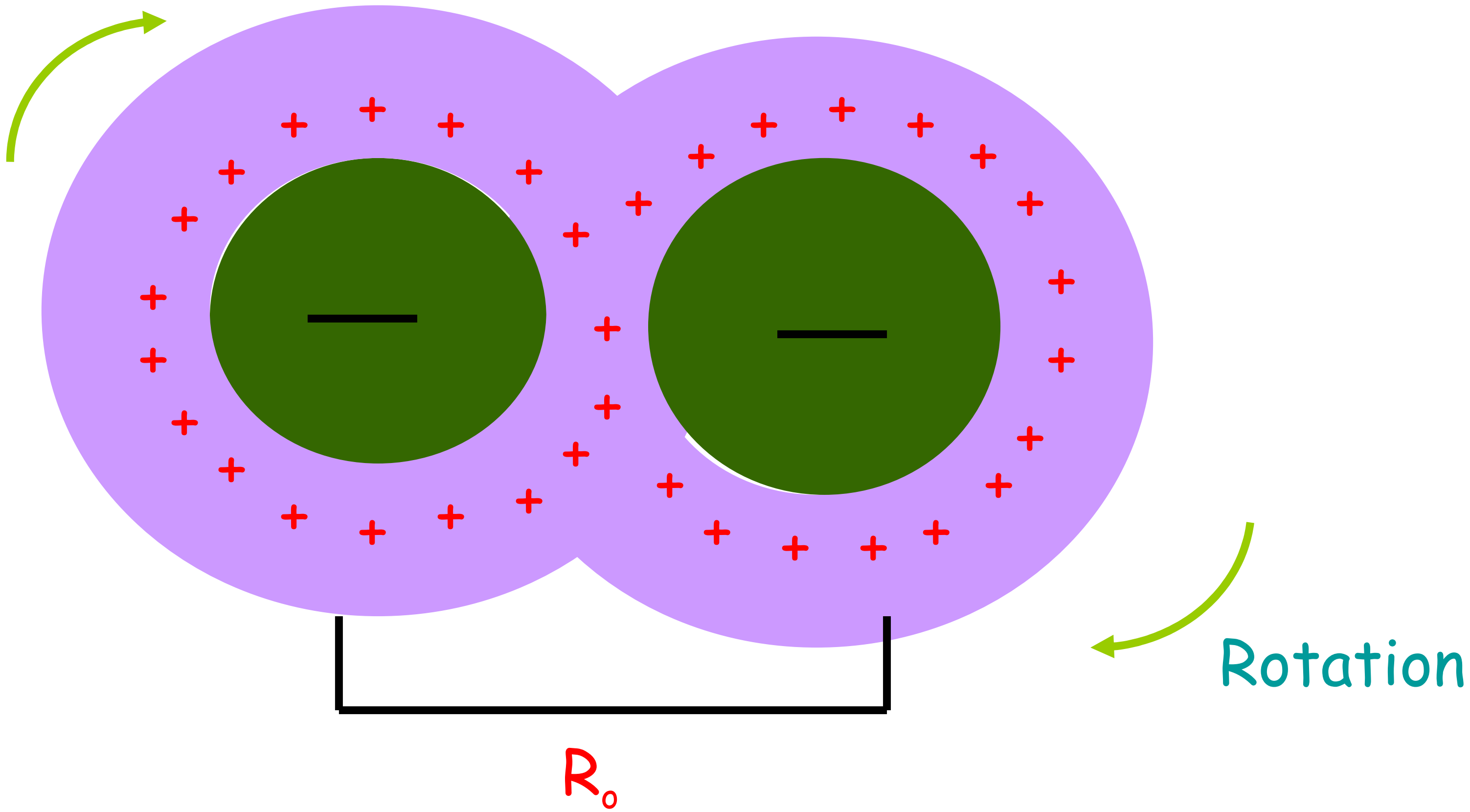
Vibrational energy

$$E_{vib} \sim \eta\omega = \left(\frac{2m_i}{M}\right)^{1/2} \frac{Z_D^{2/3}\eta^2}{m_i a_o^2}$$

Rotational energy

$$E_{Rot} \sim \frac{\eta^2}{2MR_o^2} = \left(\frac{2m_i}{M}\right)^{1/2} \frac{\eta^2}{m_i a_o^2}$$

Thus, as in the ordinary molecule, $E_{pro} > E_{vib} > E_{rot}$



Dust Molecule

Exchange Energy

Considering the weak interaction between the clouds of two dust grains and taking into account the Coulomb interaction of the protons,

$$U = -V\frac{\pi e^2 \eta^2 n^2}{2m_i T}$$

For adiabatic case

$$U_{adia} = -Ve^2\left(\frac{n}{2}\right)^{4/3}$$

For the parameters n=10^9 cm^{-3},T ≈ 300K, we obtain for |U| ~ 0,1eV

Further Suggestions

- **Stability of Dust Atom**

- For Ultra relativistic temperature

$$\nabla^2 \phi = \phi^3$$

Self Focusing and Crystallization

- Sheath Problem
- Raman spectroscopy

Conclusion

Quantum mechanical, nuclear and chemical behaviors can also be studied in Plasma Physics

This is not the end but Yet
to
explore new thoughts