

Holography and de Sitter space

The dS/dS Correspondence

“Hologravity”

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Plan of talk

- Motivation and introduction to holography
- Explicit example: AdS/CFT
- Holography of dS space¹

¹This part is based on M. Alishahiha, A. Karch and E. Silverstein, ‘Hologravity,’ JHEP **0506**, 028 (2005) and M. Alishahiha, A. Karch, E. Silverstein and D. Tong, “The dS/dS correspondence,” AIP Conf. Proc. **743**, 393 (2005)

One of our favourite equations is Bekenstein-Hawking area-entropy law

$$S = \frac{A}{4G}$$

One interesting feature of this formula is its **universality**.

It applies to all kinds of black holes

it even applies for cosmological horizons

It is universal in the sense that it is independent of the specific characteristics and composition of matter system.

Its validity is not truly universal: **It applies only when there is weakly coupled gravity**

This might put a bound on the entropy of a system with gravitational interaction.

1. G. 't Hooft, "Dimensional reduction in quantum gravity," [gr-qc/9310026](#).
2. L. Susskind, "The world as a hologram," [hep-th/9409089](#)

Assumptions

1. Spherically symmetry
 2. Weakly coupled gravity
 3. Asymptotic flat space time
- Consider an isolated matter system with mass M and entropy S_m .
 - Consider A to be the area of the smallest sphere that fits around the system.
 - To get stable system M must be less than the mass of a black hole of the same area.
 - Consider a shell with mass m such that adding to the system it can be converted to black hole.

Initial entropy

$$S_{\text{total}}^{\text{initial}} = S_m + S_{sh}$$

Final entropy

$$S_{\text{total}}^{\text{final}} = S_{\text{BH}} = \frac{A}{4G}$$

The initial entropy must not exceed the final entropy + $S_{sh} > 0$ one gets “*spherical entropy bound*”

$$S_m \leq \frac{A}{4G}$$

From thermodynamical point of view the entropy has a statistical interpretation: e^S is the number of independent quantum states on the volume surrounded by A .

The entropy bound will also put a bound on the number of degrees of freedom.

$\frac{A}{4G}$ degrees of freedom are sufficient to fully describe any stable region enclosed by a sphere of area A .

Holographic principle

A region with boundary of area A is fully described by no more than $\frac{A}{4G}$ degrees of freedom or about 1 bit of information per Planck area.

This is reminiscent of a hologram that we are familiar in optical technology in which a three-dimensional image is stored on a two-dimensional surface.

Note that it is not a straightforward projection and in fact its relation to three-dimensions is complicated.

Most of our intuition has come from AdS/CFT correspondence.

What is AdS/CFT correspondence?

The statement

A $d + 1$ dimensional theory which includes gravity is “*dual*” to a d dimensional theory without gravity.

Type IIB string theory on $AdS_5 \times S^5$ with N units of five-form flux on S^5 is *dual* to $\mathcal{N} = 4$ SYM theory in four dimensions with gauge group $U(N)$.

1. AdS_5 is a maximally symmetric solution of Einstein action with negative cosmological constant.

$$-X_0^2 - X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 = R^2$$

2. metric

$$ds^2 = \frac{U^2}{r^2} d\vec{X}^2 + \frac{R^2}{U^2} dU^2$$

$$ds^2 = R^2 (\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2)$$

3. The gauge theory lives on the boundary of the AdS_5 .
4. There is one to one correspondence between objects of gauge theory and those on gravity.

String theory side \longleftrightarrow Gauge theory side

String coupling \longleftrightarrow N

Curvature \longleftrightarrow 't Hooft coupling

$SO(2, 4) \times SO(6)$ \longleftrightarrow SC – group \times R – symmetry

fields, strings, \longleftrightarrow Gauge inv. operators

Partition functions \longleftrightarrow Generating functions

Consider field ϕ with mass m such that

$$\phi|_{\text{boundary}} = \phi_0$$

therefore

$$W[\phi_0] = e^{S[\phi]} = \langle e^{\int \phi_0 \mathcal{O}} \rangle_{SYM}$$

and dimension of operator \mathcal{O} , Δ is given in terms of d and m .

If we know one side, one may get information for the other side

If gravity can be understood holographically, what would be the dual theory for other backgrounds?

What about the de Sitter space?

What is de Sitter space?

de Sitter space is the maximally symmetric solution of Einstein equations with **positive** cosmological constant.

In fact the dS_d space can be realized as a hypersurface described by the following algebraic equation in flat $d + 1$ dimensional Minkowski space

$$-X_0^2 + X_1^2 + \dots + X_d^2 = l^2$$

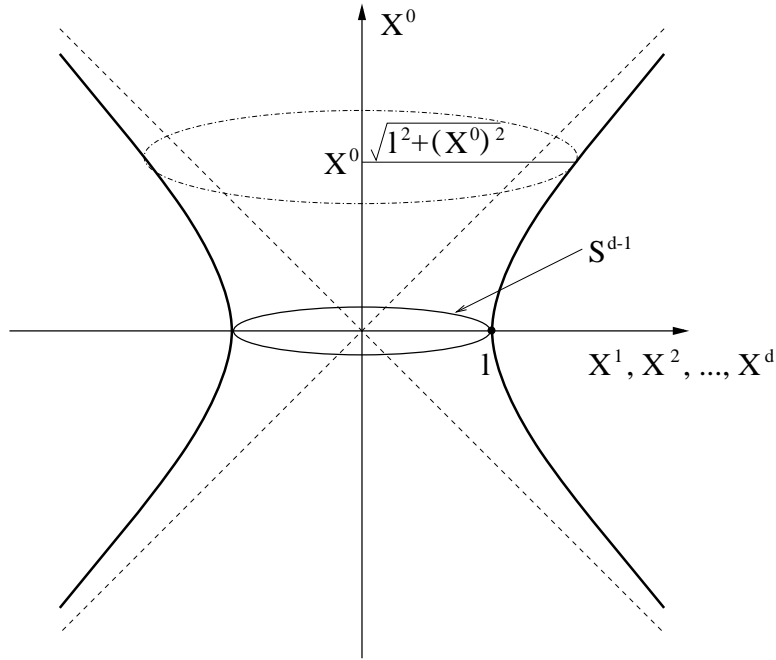
The dS metric is the induced metric from the flat space

$$dS^2 = \eta^{AB} dX_A dX_B |_{\eta^{AB} X_A X_B = l^2}$$

Comparing with the Einstein equation we had which was written in terms of the cosmological constant one finds

$$\Lambda = \frac{(d-1)(d-2)}{2l^2}$$

Therefore the dS_d in the flat $d + 1$ dimensional Minkowski space is a hyperboloid.

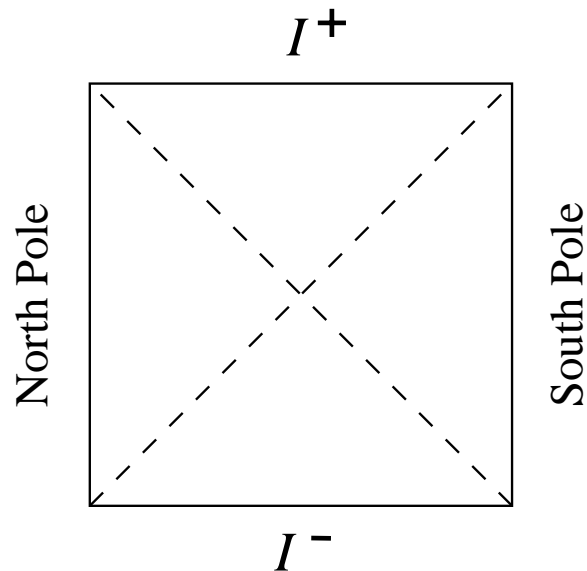


$$dS^2 = -d\tau^2 + l^2 \cosh^2 \frac{\tau}{l} d\Omega_{d-1}^2$$

In this coordinates dS_d space looks like a $d - 1$ sphere which starts out infinity large at $\tau = -\infty$ then shrinks to its minimum finite size that $\tau = 0$ and then grows again to infinite size at $\tau = \infty$.

$$dS^2 = -dt^2 + e^{-2t/l} dx_i dx^i,$$

$$dS^2 = -\left(1 - \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{1 - \frac{r^2}{l^2}} + r^2 d\Omega_{d-2}^2$$



- 1) North and South poles are timelike line.
- 2) Every point in the interior represents an S^{d-2} .
- 3) A horizontal slice is an S^{d-1} .
- 4) I^- and I^+ are past and future null infinity. They are the surfaces where all null geodesics originate and terminate.
- 5) The dashed lines are the past and future horizons of an observer at the south pole.

Therefore no single observer can see entire spacetime.

Why de sitter space?

- The recent astronomical observations indicate that the cosmological constant in our universe is not zero.

Not only it is not zero but also its contribution is quite important and in fact it is responsible for almost 73% of the energy of the universe (dark energy).

This means our universe might currently be in the de Sitter phase.

- Beside from this observation, another motivation to study de Sitter space comes from inflation era in which we assume that the universe was also described by de Sitter phase.

- Another interesting feature of dS space is that it has cosmological horizon in which one can associate a temperature and entropy

$$S = \frac{A}{4G}$$

Like black hole, we would like to understand this entropy for dS as well.

Why dS is difficult?

1) dS space is inconsistent with supersymmetry: there is no supergroup that includes the isometries of dS space and has unitary representation.

2) We have not been able to embed it in string theory (up to KKLT model).

3) Horizon is observer dependent: difficult to see where the quantum microstates we would like to count are in fact supposed to be.

From what we have learned in AdS/CFT correspondence one may hope that some kind of holography can also be applied here and could help us to understand the quantum gravity on dS.

There is a naive observation:

Consider a AdS space with radius R , under $R \rightarrow iR$ one gets

$$\begin{array}{ccc} \Lambda & \longrightarrow & -\Lambda \\ AdS & \longrightarrow & dS \\ SO(2, d) & \longrightarrow & SO(1, d + 1) \end{array}$$

Gravity on dS is dual to a Euclidean CFT.

One can make this statement more precise which is in fact what is known as dS/CFT correspondence.

Consider the Brown-York stress tensor $T^{\mu\nu}$ for a spacetime which is asymptotically dS. One may ask the following questions:

1. What would be the boundary conditions for the metric if we want the stress tensor to be finite?
2. What is the most general diffeomorphism which preserves this boundary conditions?

The first question can be answered by perturbing dS space and computing $T^{\mu\nu}$ and then one may answer to the second question which will be

The conformal group of the $(d - 1)$ -dimensional Euclidean space.

This is one of the main hints of the dS/CFT correspondence which says

Quantum gravity on dS_d is dual to a $(d - 1)$ -dimensional Euclidean conformal field theory residing on the past boundary I^- of dS_d .

This CFT may be non-unitary!

We note however that in studying this correspondence one uses the dS in the planer metric. So it is nature to ask:

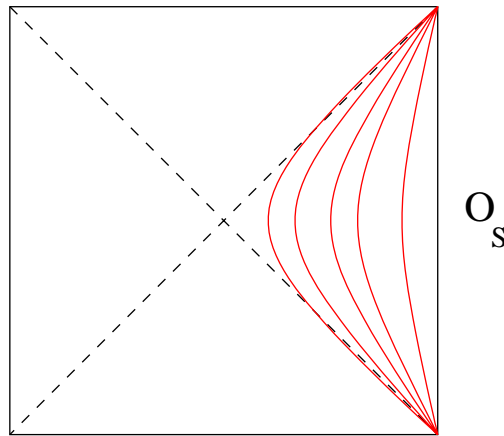
How would the holography work if we picked up another coordinate system?

Statement of de Sitter Holography

The dS_d static patch is dual to two conformal field theories on dS_{d-1} cut off at an energy scale $1/R$ and coupled to each other as well as to $(d-1)$ -dimensional gravity.

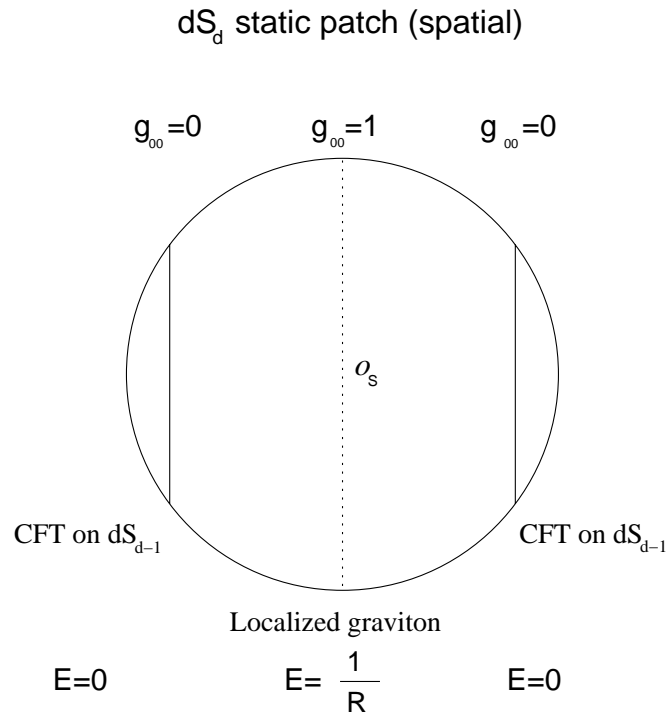
The static patch of d -dimensional dS space with radius R can be foliated by dS_{d-1} slices

$$ds^2 = \sin^2\left(\frac{w}{R}\right) ds_{dS_{d-1}}^2 + dw^2$$



The warp factor $\sin^2(w/R)$ is maximal with the finite value at central slice $w = \pi R/2$ and dropping monotonically on each side until reaches zero at the horizon $w = 0, \pi$.

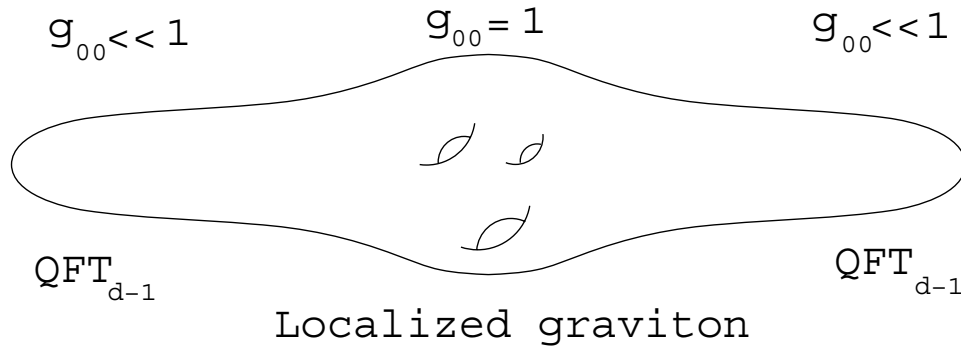
The region near horizon (low energy in static patch) is isomorphic to d -dimensional AdS space foliated by dS_{d-1} slices and hence constitutes a CFT on dS_{d-1} at low energy.



- D-brane probes of this region exhibit the same physics: Both are equivalent to CFT on dS_{d-1} for energy $0 < E \ll \frac{1}{R}$.
- Probes constructed from bulk gravitons range from energy 0 up to $1/R$ and upon dimensional reduction their spectrum exhibits the mass gap expected of $d - 1$ dimensional CFT on dS .
- Dimensionally reducing to $d - 1$ dimensional effective field theory also yields a finite $d - 1$ dimensional Plank mass \rightarrow Lower dimensional theory has also gravity.

The picture

Written in a dS_{d-1} slicing, dS_d has the form of a Randall-Sundrum system.

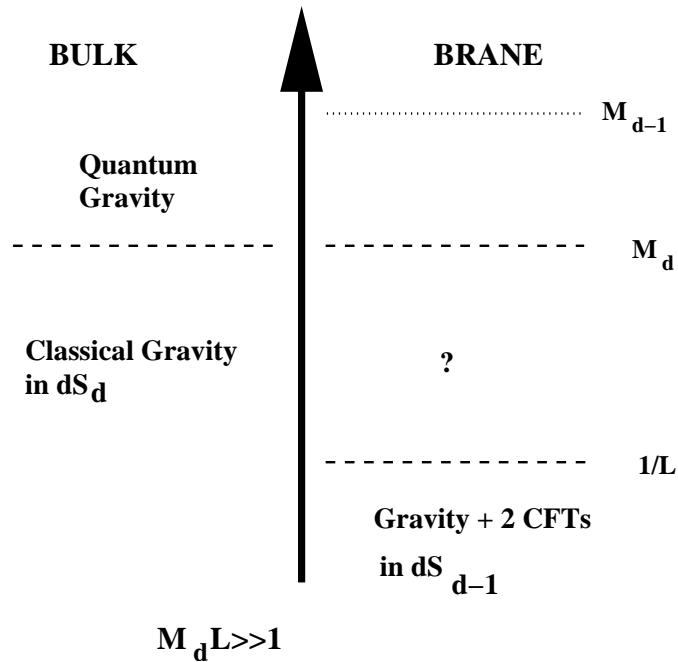


At higher energies $E \rightarrow \frac{1}{R}$ AdS and dS differ:

- In AdS, the warp factor diverges toward the UV $\rightarrow d - 1$ dimensional gravity decouples.
- In dS, the warp factor is bounded \rightarrow a dynamical $d - 1$ dimensional graviton.

In the RS construction one truncates the warp factor at a finite value of the radial coordinate by including a brane source with extra degree of freedom. In the dS case, the additional brane source is unnecessary.

A smooth UV brane at which the warp factor turns around is built in to the geometry.



- On d -dimensional gravity side, one has local effective field theory up to $M_d \gg 1/R$.
- Above M_d quantum gravity effects become important in the bulk and one has to UV complete the system.
- Using gravity side one can study the $d - 1$ dimensional theory in the range of energy $1/R < E < M_d$.

One can use AdS/CFT correspondence to study some properties of this duality.

dS slicing of dS:

$$ds_{dS_d}^2 = \frac{R^2}{\cosh^2(z)} (ds_{dS_{d-1}}^2 + dz^2)$$

dS slicing of AdS:

$$ds_{AdS_d}^2 = \frac{R^2}{\sinh^2(z)} (ds_{dS_{d-1}}^2 + dz^2)$$

So

$$ds_{AdS_d}^2 = \frac{1}{\tanh^2(z)} ds_{dS_d}^2$$

We can use this to map the physics of dS to dynamics in AdS, albeit with unusual actions. Namely this leads to

- Scalars with position dependent masses.
- Gravity with a position dependent Newton constant.

By applying the AdS/CFT dictionary to the resulting system, this allows us to make a direct comparison of UV behavior of the $d - 1$ dual of dS_d to the UV behavior of a strongly coupled CFT.

g_{mn}	\rightarrow	$f^2 g_{mn}$
X	\rightarrow	$f^{-\frac{d-2}{2}} X$
$\sqrt{-g}$	\rightarrow	$f^d \sqrt{-g}$
$-\sqrt{-g}(\partial X)^2$	\rightarrow	$-\sqrt{-g}(\partial X)^2 - \sqrt{-g} \frac{(d-2)}{2} X^2 (\nabla^2 \omega)$ $-\sqrt{-g} \frac{(d-2)^2}{4} X^2 (\nabla \omega)^2$
$\sqrt{-g} R$	\rightarrow	$f^{d-2} \sqrt{-g} (R - 2(d-1)(\nabla^2 \omega)$ $-(d-2)(d-1)(\nabla \omega)^2)$
$-\sqrt{-g} \xi R X^2$	\rightarrow	$-\sqrt{-g} \xi R X^2 + 2\sqrt{-g} \xi (d-1)(\nabla^2 \omega)$ $+\sqrt{-g} \xi (d-2)(d-1)(\nabla \omega)^2$
$-2\sqrt{-g} \Lambda$	\rightarrow	$-2f^d \sqrt{-g} \Lambda$

Table 1: Transformations under conformal rescaling;
 $f = \tanh z$, $\omega = \log f$

Under this conformal map one has:

- The bulk action for a free, massive scalar field in dS_d

$$S = \int d^d x \sqrt{-g} \left(-(\partial_\mu X)^2 - (M^2 + \xi R) X^2 \right)$$

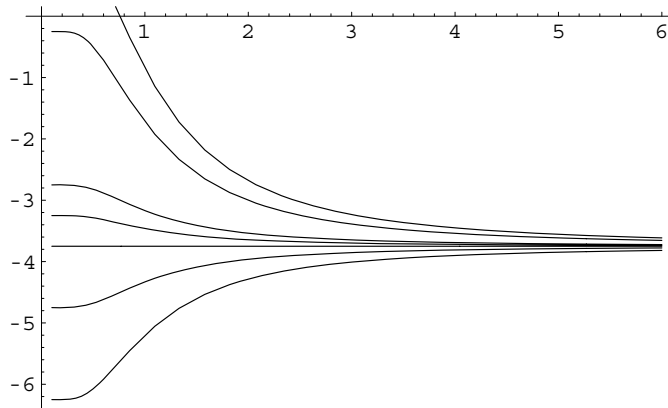
maps to scalar field in AdS with mass

$$M_{\text{total}}^2 = \tanh^2(z) (M^2 + \xi d(d-1)) - \frac{d(d-2)}{4} (1 + \tanh^2(z))$$

where we used $R = -d(d-1)$ for the AdS_d .

In the UV ($z = 0$) the original dS mass term M scales to zero, as does the original ξ term. Instead we get the universal result

$$M_{\text{total}}^2 = -\frac{d(d-2)}{4} \quad \text{for all } \xi, M.$$



That is we get a conformally coupled scalar in AdS, independent of what values of the parameters M and ξ we started with in dS! The corresponding UV dimension of the dual operator is

$$\Delta_O = \left\{ \begin{array}{l} \frac{d}{2} \\ \frac{d}{2} - 1 \end{array} \right.$$

This ensures that the $\langle OO \rangle$ two point function for the second choice reduces to the usual $\frac{1}{|x|^{d-2}}$ behavior of a scalar field in d dimensions.

- Starting with the Einstein-Hilbert action in dS

$$S = M_d^{d-2} \int d^d x \sqrt{-g} (R - 2\Lambda)$$

where $\Lambda = \frac{1}{2}(d-1)(d-2)$, we get a new gravitational action in AdS

$$S = \int d^d x \sqrt{-g} (M_d f)^{d-2} (R + (d-2)(d-1)(\nabla\omega)^2 - 2f^2\Lambda)$$

Close to the boundary the graviton looks like a flat space graviton! The possible boundary behaviors are z^0, z^1 , as opposed to z^0, z^4 in AdS. Still this couple to energy-momentum tensor with dimension $d-1$.

Conformal Anomaly

AdS/CFT instructs us to evaluate the bulk action on a given solution in order to calculate the boundary partition function for a given boundary metric. This quantity has divergences due to the infinite volume of AdS. One needs to add local counterterms.

However in even boundary dimensions (odd bulk dimension d) there are in addition $\log(z)$ terms and they represent the conformal anomaly.

For the standard Einstein Hilbert action on the dS_{d-1} sliced AdS_d background we get

$$S_{\text{on-shell}} = \frac{-2(d-1)}{16\pi G_N} \int \frac{dz}{\sinh^d(z)}$$

Expanding in powers of z around $z = 0$ one finds

$$\begin{aligned} \int \frac{dz}{\sinh^3(z)} &= -\frac{1}{2z^2} - \frac{\log(z)}{2} + \mathcal{O}(z^0) \\ \int \frac{dz}{\sinh^5(z)} &= -\frac{1}{4z^4} + \frac{5}{12z^2} + \frac{3}{8}\log(z) + \mathcal{O}(z^0) \\ \int \frac{dz}{\sinh^7(z)} &= -\frac{1}{6z^6} + \frac{7}{24z^2} - \frac{259}{720z^2} - \frac{5}{16}\log(z) + \mathcal{O}(z^0) \end{aligned}$$

The log terms give the conformal anomaly evaluated on dS_2 , dS_4 and dS_6 respectively.

Now let us repeat the same exercise for the gravitational action with position dependent G_N . The position dependence of M_d is $M_d(z) = \tanh(z)M_d$ which gives an extra factor of $\tanh^{d-2}(z)$. Up to terms that remain finite as $z \rightarrow 0$

$$\begin{aligned} S_{\text{on-shell}} &= \int \tanh^{d-2}(z) \frac{dz}{\sinh^d(z)} \\ &= \int \frac{1}{\sinh^2(z) \cosh^d(z)} dz = \frac{-1}{z} + \mathcal{O}(z) \end{aligned}$$

For all d the only divergent term is a universal $-\frac{1}{z}$ and there are no logarithms. The conformal anomaly vanishes.

One possible interpretation:

- Lower dimensional gravity screens the central charge to be zero, just like is well known from 2d gravity on string theory worldsheets. In this scenario one does not even need a conformal field theory beyond scales $1/R$ since the gravitational dressing will also make any FT a CFT.
- In the same spirit the universal UV dimension of the scalar fields can be understood as gravitational dressing.

Conclusions

- Holography might lead to a fundamental theory which includes quantum gravity.
- String theory has provided us an explicit example of holography: AdS/CFT correspondence.
- This not only might help us to understand quantum gravity and black hole, but also it might provide a framework to study non-perturbative gauge theory (QCD).
- One may also study quantum gravity on dS using holography.
- So far there are two proposed holography for dS space: dS/CFT and dS/dS.