

Neutrinos

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Neutrino was the first particle postulated by a theoretician: **W. Pauli in 1930** to save conservation of energy and A. M. in β decays.

Ever since its direct observation in **1950's** by Reines and Cowan, this elusive particle which has almost no interaction with matter, has contributed to some of the most important discoveries in Physics.

7 Nobel Prizes since 1950 involve neutrinos in one way or other.

Certainly one of the most exciting areas or research at present is neutrino physics. Neutrinos are fantastically numerous in the universe and so to understand the universe we must understand neutrinos.

What is known about neutrinos.

- Neutrinos are elementary particles with spin $\frac{1}{2}$, electrically neutral and obey Fermi-Dirac Statistics.
- Neutrinos are part of stellar dynamics:

The energy of the sun is generated through the fusion:



There are about $7 \times 10^9 \text{ cm}^{-2} \text{ sec}^{-1}$ neutrinos from the sun reaching the earth.

If neutrinos were not there, the stars would not shine and we would not be here.

- Neutrinos occur in three flavors:

$$\nu_e, \nu_\mu, \nu_\tau \quad \left(\begin{array}{|c|} \hline \nu_e \\ \hline e \\ \hline \end{array} \right), \left(\begin{array}{|c|} \hline \nu_\mu \\ \hline \mu \\ \hline \end{array} \right), \left(\begin{array}{|c|} \hline \nu_\tau \\ \hline \tau \\ \hline \end{array} \right)$$

- All neutrinos detected are left-handed i.e. their spin points in the opposite direction from their momentum. All anti-neutrinos are right-handed.
- Neutrinos “oscillate” from one specie to another with a high probability. This means that neutrino produced in a well-defined weak eigenstate ν_α can be detected in a distinct weak eigenstate ν_β .

Such a flavor change is observed and the simplest way to explain this phenomenon is to postulate that neutrinos have distinct non-zero mass, and the neutrino mass eigenstates are different from the neutrino weak eigenstates, the later are generally coherent superposition of the former

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

The neutrino will undergo oscillations as they propagate and the oscillating probability $P_{\alpha\beta}$ is a function of the propagation distance L , neutrino mass difference $\Delta m_{ij}^2 = m_j^2 - m_i^2$

neutrino energy E_ν and the elements of the lepton mixing matrix $U_{\alpha i}$ e.g. for two flavor conversion.

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left[1.27 \Delta m^2 / E_\nu \right] L$$

where L is measured in meters and $\Delta m^2 = m_1^2 - m_2^2$ in units of eV^2 while the neutrino energy E_ν is measured in MeV and θ is the mixing angle in 2×2 scenario.

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The above result illustrate the quantum mechanical phenomenon of interferometry which provides a sensitive method to probe extremely small effects. For interferometry to work, one needs;

- i. a coherent source: luckily there are many coherent sources of neutrinos: the sun, cosmic rays, reactors etc.
- ii. interference: luckily there are large mixing angles to make interference possible.
- iii. large baseline to enhance the tiny effects: again luckily many baselines are available: size of the sun, that of the earth etc.

Thus nature is very kind to provide all ingredients for neutrinos oscillations (interferometry) to work, thereby

providing us a unique tool to study physics at very high energy scales.

For 3 flavors, it is customary to parameterize mixing matrix elements $U_{\alpha i}$ (not all independent) by three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one complex phase δ

$$\frac{|U_{e2}|^2}{|U_{e1}|^2} = \tan^2 \theta_{12}, \quad \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = \tan^2 \theta_{23}$$

The mass eigenstate $|v_i\rangle$ ($i = 1, 2, 3$) has a well defined mass m_i and it is customary to order the mass eigenvalues such that $m_1^2 < m_2^2$, $\Delta m_{12}^2 < |\Delta m_{13}^2|$

$$\Delta m_{13}^2 > 0 \Rightarrow m_3^2 > m_1^2 \quad \text{normal mass hierarchy}$$

$$\Delta m_{13}^2 < 0 \Rightarrow m_3^2 < m_1^2 \quad \text{inverted mass hierarchy}$$

Detailed combined analysis of all neutrino data are consistent at 3σ level with

$$\sin^2 2\theta_{23} \geq 0.92, \quad 1.2 \times 10^{-3} \text{eV}^2 \leq |\Delta m_{13}^2| \leq 4.8 \times 10^{-3} \text{eV}^2$$

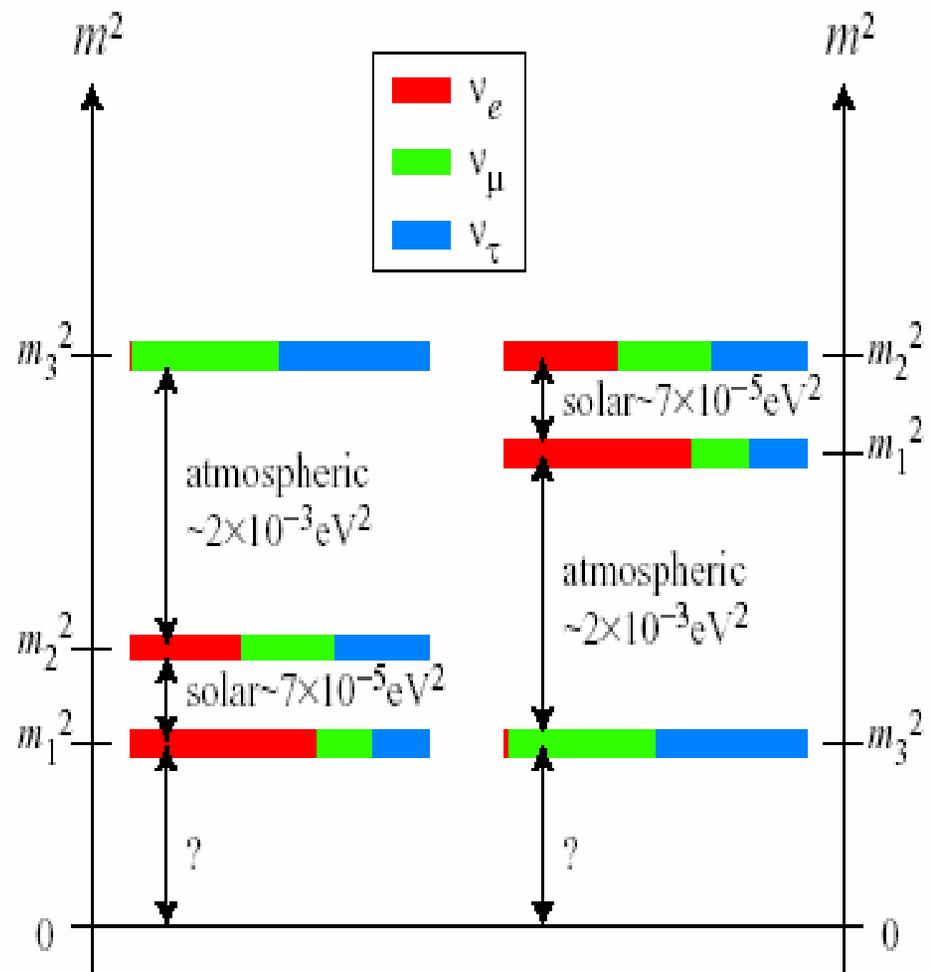
$$0.70 \leq \sin^2 2\theta_{12} \leq 0.95, \quad 5.2 \times 10^{-5} \text{eV}^2 \leq |\Delta m_{12}^2| \leq 9.5 \times 10^{-5} \text{eV}^2$$

$$\sin^2 \theta_{13} \leq 0.0047$$

Currently, there is no constraint on CP-odd phase δ or on the sign of Δm_{13} .

Since the oscillation data are only sensitive to mass squared differences, they allow for 3 possible arrangements of the different mass levels.

Degenerate neutrinos
i.e. $m_1 \approx m_2 \approx m_3$



The most important conclusion one draws is that “NEUTRINOs HAVE MASS”. However oscillation experiments do not tell us about overall scale of masses, but they are exceedingly tiny.

The most straightforward limit on the absolute value of neutrino masses is obtained by looking for structure near the end point of the electron energy spectra in tritium β -decay. These searches reveal

$$m_{\beta} = \sqrt{\sum_i |U_{ei}|^2 m_i^2} \leq 2.2 \text{ eV}$$

The Spectrum Scale of ν -mass from Present Goal

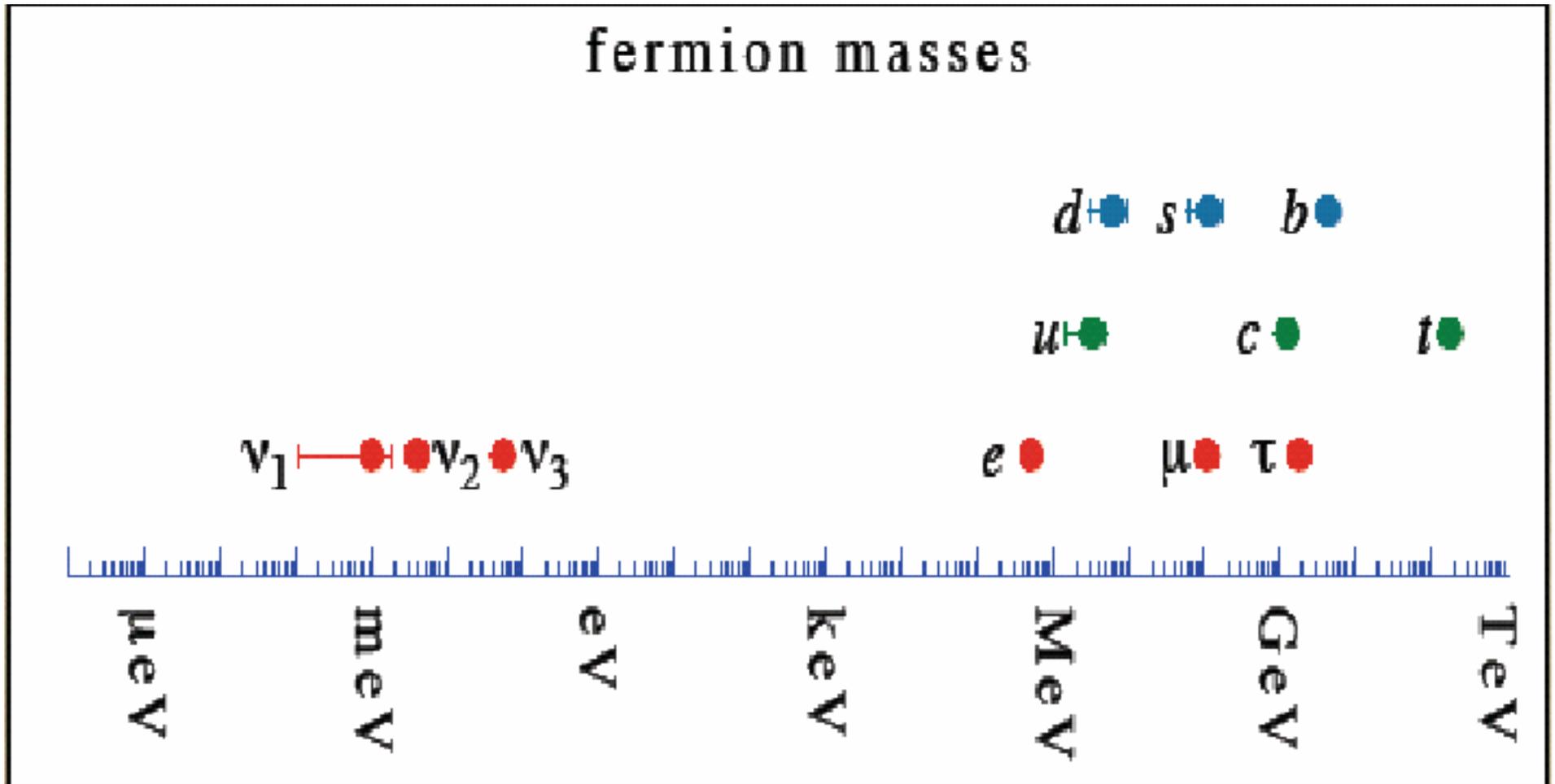
Tritium β decay $\sum_i m_{\beta} \leq 2 \text{ eV} \quad \sim 0.2 \text{ eV}$

Cosmology $\sum_i m_i \leq 0.69 \text{ eV} \quad \sim (0.05-0.1) \text{ eV}$

$$\beta\beta 0\nu \quad m_{\beta\beta} = \sum_i |U_{ei}^2 m_i| \quad \sim 0.02 \text{ eV}$$
$$\leq 0.3 \text{ eV}$$

By combining all bounds it is safe to say that all neutrinos weigh less than 1eV.

Fig. Depicts the value of all known fundamental fermions:



One sees that the gap between neutrino masses and the lightest charged fermion (m_e) is deserted in contrast to that between m_e and m_t which is populated.

Further

$$\frac{(m_\nu)_{\max}}{m_e} < 2 \times 10^{-6}$$

which need to be understood.

Origin of Neutrino Mass

Neutrino occurs in one helicity state (left handed). This together with lepton number conservation implies $m_\nu = 0$. However, there is no deep reason that it should be so. There is no local gauge symmetry and no massless gauge boson coupled to lepton number L , which therefore expected to be violated. Thus one may expect a finite mass for neutrino.

However SM conserve L , nor does it contain any chirally right-handed neutral fields, but only left-handed ones, ν_L .

If one allows right-handed neutrinos N_i which are $SU(2) \times U(1)$ singlets, then one can write Yukawa interactions

$$\mathcal{L}_{\text{new}} = \mathcal{L}_{\text{SM}} - h_{\alpha i} \bar{L}_\alpha H N_i + h.c.$$

L is left handed lepton doublet, H is the Higgs doublet. After electroweak symmetry breaking,

$$\mathcal{L}_{\text{mass}} = -h \langle H \rangle \bar{\nu}_L N_R + h.c.$$

This does not mix neutrinos and antineutrinos, so it conserves L .

The neutrino mass matrix is

$$m_\nu = h \langle H \rangle$$

The magnitude of neutrino mass requires $h \leq 10^{-12}$, at least 6 orders of magnitude smaller than electron Yukawa couplings.

A natural explanation of the smallness of m_ν is not contained in the above equation.

Majorana Neutrinos

An interesting question about the intrinsic nature of neutrinos, raised by discovery of neutrino mass is

Are neutrinos their own antiparticles?

i.e. for given helicity h

$$\bar{\nu}_i(h) = \nu_i(h)$$

N_R being electroweak isospin singlet, all the SM principles, including electroweak isospin conservation allow

“Majorana mass term” $\mathcal{L}_M = -M\bar{N}_R^c N_R + h.c.$

N_R^c is the charge conjugate of N_R .

Converts N to \bar{N} \Rightarrow Does not conserve L .

The most economical way to add neutrino mass to the SM is that neutrinos have Majorana mass arising from $\Delta L=2$ non-renormalizable interactions of the form

$$L_{eff} = \frac{G}{M} LHLH$$

After electroweak symmetry breaking i.e. replacing the Higgs field by its expectation value $\langle H \rangle = v$,

$$L_{eff} \rightarrow \frac{G}{M} (L \langle H \rangle)(L \langle H \rangle)$$

$$= m_\nu \nu\nu$$

nothing but neutrino mass

$$m_\nu = \frac{G}{M} v^2$$

Such an effective interaction can be generated

$$L_Y = \bar{L}_i H h_{ij} e_{Rj} + \bar{L} H h^*_{ij} N_{Rj} - \frac{1}{2} \bar{N}^c_R M N_R + h.c.$$

$i, j=1,2,3$ for 3 leptons families. The lepton number violating term is introduced by the third term. M is Majorana mass matrix while h_{ij} are Yukawa couplings. After spontaneous symmetry breaking, Dirac mass term is generated $(m_D)_{ij} = h_{ij} v$ assumed to be small compared to M . Light neutrino mass matrix M_ν arise from diagonalizing the 6×6 neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}$$

and takes the seesaw form

$$m_{\nu} = -m_D^T M^{-1} m_D$$

$$m_{N_i} = M_i$$

This matrix has an eigenvalue

$$m_{\nu_i} \sim \frac{m_D^2}{M} \approx \frac{v^2}{M} \ll m_l$$

by requiring the existence of scale M , associated with new physics. With $v = 175 \text{ GeV}$, the above number is of the order needed to explain neutrino $m_{\nu} \approx 0.045 \text{ eV}$ anomaly for $M \sim 10^{15} \text{ GeV}$ or so, not much different from GUT scale and other scales which have been proposed for new physics.

If neutrinos are Majorana particles, the 3×3 leptonic matrix U may contain 3 CP violating Majorana phase ϕ_i associated with neutrino (self conjugate) mass eigenstate ν_i .

In their presence,

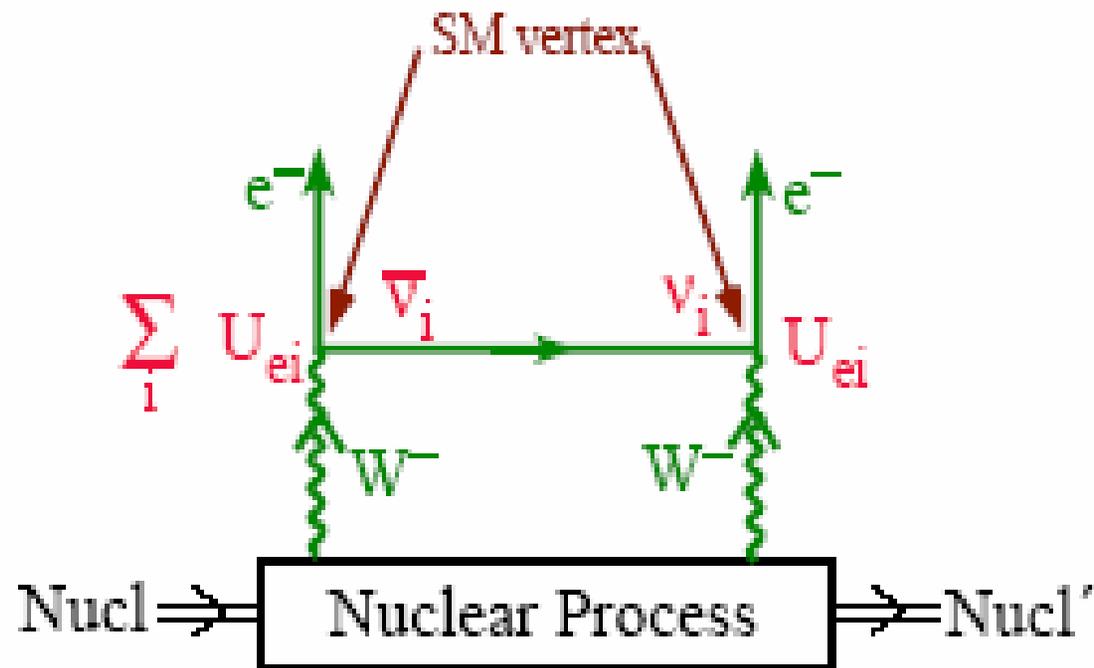
$$U_{\alpha i}^0 \rightarrow U_{\alpha i} = U_{\alpha i}^0 e^{i\phi_i} \quad ; \text{ all } \alpha .$$

Can one test Majorana character of neutrino?

SM conserves lepton number L , so that L non conservation we seek can come only from Majorana mass term and as such will be

challenged by smallness of m_ν .

Search of neutrinoless double β -decay is the only one approach that shows considerable promise of meeting this challenge.



The above process does not exist if $\bar{\nu}_i \neq \nu_i$, helicity of $\bar{\nu}_i$ cannot be exactly +1 but contain a small piece, of order m_i/E_{ν_i} having helicity -1.

Thus contribution of ν_i exchange is proportional to m_i .

$$A m p [0 \nu \beta \beta] \propto \left| U_{ei}^2 m_i \right| \equiv m_{\beta\beta}$$

$$|m_{\beta\beta}| = \left| \cos\theta_{13} \left(|m_1| e^{-2i\phi_1} \cos^2\theta_{12} + |m_2| e^{-2i\phi_2} \sin^2\theta_{12} \right) + \sin^2\theta_{13} |m_3| e^{-2i\delta} \right|$$

Current experiments rule out $m_{\beta\beta} \geq 1\text{eV}$ and the present upper bound is $\leq 0.3\text{eV}$. A controversial and yet-to-be confirmed analysis of ^{76}Ge decay data by Heidelberg-Moscow group claims $m_{\beta\beta}$ lies between 0.11eV to 0.58eV .

If confirmed this result is of fundamental importance first indication of lepton number violation and that Majorana neutrino can exist in nature.

Neutrino Mass Models

Phenomenological models derived by the data are to involve some peculiar feature of neutrino mixing such as

- Some or (all) neutrino masses could be quasi-degenerate in absolute value.
- $|U_{e3}| \ll$ all other entries in the neutrino mixing matrix U .
- $|U_{\mu 3}| \simeq |U_{\tau 3}| \Rightarrow$ maximal atmospheric mixing.

Such models in turn give predictions to parameters not measured so far. The good thing about such models is that they are falsifiable.

Broadly speaking neutrino mass matrix which are consistent with the mass-squared mass difference are exemplified as follows

Texture	Hierarchy	$ U_{e3} $
$\frac{\sqrt{\Delta m_{13}^2}}{2} \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$	Normal (N)	$\sqrt{\frac{\Delta m_{12}^2}{\Delta m_{13}^2}}$
$\sqrt{\Delta m_{13}^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \text{small}$	Inverted (I)	$\simeq 0$
$m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{small}$	Quasi-Degenerate (QD)	$\frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$

R. N. Mohapatra et al. hep-ph/0412099

Measurement of whether $|U_{e3}|^2 \gg 0.01$ and/or $|\cos\theta_{23}| \gtrsim 0.01$ will allow us to determine the best path to follow as far as understanding of neutrino mass and lepton mixing is concerned.

Predictions for Maximal and Minimal value of $\langle m \rangle_{\text{eff}}$ in unit of MeV neutrinoless double β decay.

$$|\Delta m_{13}^2| = 2.6 \times 10^{-3} \text{eV}, m_0 = 0.2 \text{eV} \quad \text{for QD}$$

$\sin^2 \theta_{13}$	$\langle m \rangle_{eff}^{NH}_{max}$	$\langle m \rangle_{eff}^{IH}_{min}$	$\langle m \rangle_{eff}^{IH}_{max}$	$\langle m \rangle_{eff}^{QD}_{min}$
0.0	2.6	19.9	50.5	79.9
0.02	3.6	19.5	49.5	74.2
0.04	4.6	19.1	48.5	68.5

Except for NH, the next generation experiments are supposed to be able to probe the above values.

LEPTOGENESIS

Understanding the origin of matter i.e.

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6 \pm 3) \times 10^{-10}$$

is one of the fundamental questions of Cosmology.

The answer may come from Particle Physics. Three ingredients are necessary to generate η , the observed Baryon asymmetry of the Universe.

- i. Baryon number B violation
- ii. CP violation
- iii. Departure from thermal equilibrium

In SM, B and L symmetries hold at classical level.
However non perturbation quantum effects $\Rightarrow B+L$
violation although $B-L$ is preserved.

However the phase with quark mixing matrix could not
have produced near enough CP violation to explain η .

As a result there is considerable interest that excess
resulted from Leptogenesis.

In see-saw mechanism each light neutrino is
accompanied by heavy neutrino N .

Both are Majorana particles. Thus there is a CP violation
coming from Majorana phases.

The CP violation leads to unequal rates for the leptonic decays

$$N \rightarrow l^+ + \text{Higgs}^- \quad \text{and} \quad N \rightarrow l^- + \text{Higgs}^+$$

We shall restrict our discussion to the case of hierarchical Majorana neutrino masses, $M_1 \ll M_2, M_3$ so that if the interactions of $N_1 = N$ are in thermal equilibrium when N_2 and N_3 decay, the asymmetry produced by N_2 and N_3 can be erased before N_1 decays. The asymmetry is then generated by the out of equilibrium CP violating decays of $N \rightarrow \ell H$ versus $N \rightarrow \bar{\ell} \bar{H}$ at the temperature $T \sim M \equiv M_1 \ll M_2, M_3$ where T is the temperature of thermal bath after inflation

A CP asymmetry

$$\mathcal{E}_1 = \frac{\Gamma(N_1 \rightarrow \ell_i H) - \Gamma(N_1 \rightarrow \bar{\ell}_i H^*)}{\Gamma(N_1 \rightarrow \ell_i H) + \Gamma(N_1 \rightarrow \bar{\ell}_i H^*)}$$

in the decay produces net symmetry of SM leptons. This asymmetry is partially transformed into a baryon asymmetry by non-perturbative B+L violation

$$Y_B = CY_L = C\kappa \frac{n}{s} \mathcal{E}_1$$

where $\kappa \leq 1$ is an efficient factor which can be obtained through solving the Boltzman equation $n/s \sim 10^{-3}$

is the ratio of the N_1 equilibrium density to the entropy density. $C \sim 1/3$ tells us what fraction of lepton asymmetry is converted into baryon asymmetry, to B+L violation processes.

$$Y_B = \eta \left(\frac{n_\gamma}{s} \right) \approx \frac{1}{7} \eta$$
$$\approx \frac{1}{7} (6 \pm 3) \times 10^{-10}$$

Can be explained if $\epsilon_1 \geq 10^{-6}$.

Conclusion

To conclude various neutrino mass patterns and corresponding neutrino mass matrix types are possible. Further the absolute value of neutrino mass is not yet determined. However, one thing is certain that neutrinos are providing an evidence for new physics but the scale of new physics is not yet pinned down. The heavy right handed neutrinos at new physics scale may provide an explanation for baryogenesis through leptogenesis. If past is of any guide, neutrinos will enrich physics still further.

Thanks!