Generalized Parton Distributions
Recent Progress
(Mostly a summary of various talks at SIR2005@Jlab in May 2005

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Factorisation:
$Q^2$ large, $-t < 1$ GeV$^2$
What is a GPD?

- It is a proton matrix element which is a hybrid of elastic form factors and Feynman distributions.
- GPDs depend upon:
  - $x$: fraction of the longitudinal momentum carried by struck parton
  - $t$: $t$-channel momentum transfer squared
  - $\xi$: skewness parameter (a new variable coming from selection of a light-cone direction)
  - $Q^2$: probing scale
DVCS cannot separate u/d quark contributions.

\[ M = \rho/\omega \text{ select } H, E, \text{ for u/d flavors} \]
\[ M = \pi, \eta, K \text{ select } H, E \]
Formal definition of GPDs:

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' \mid q(-\frac{1}{2} \lambda n) \gamma^+ q(\frac{1}{2} \lambda n) \mid p \rangle = H(x, \xi, t) \bar{u} \gamma^+ u + E(x, \xi, t) \bar{u} \frac{i\sigma^{+\nu} q_\nu}{2M} u \]

- \( x_i \) and \( x_f \) are the momentum fractions of the struck quark, and \( x = \frac{1}{2} (x_i + x_f) \).
- \( \xi = (x_f - x_i)/2 \) is skewness. Depends on lightcone direction.
- \( \int dx H(x, \xi, t) = F_1(t) \)
- \( \int dx E(x, \xi, t) = F_2(t) \)
Relation of GPDs to Angular Momentum

Generalized form factor and quark angular momentum:

\[ \langle P' | T^{\mu\nu}_{q,g} | P \rangle = \bar{U}(P') \left[ A_{20}^{q,g}(t) \gamma^{(\mu P^{\nu})} + B_{20}^{q,g}(t) \frac{P^{(\mu i_{\sigma}^{\nu})\alpha} \Delta_{\alpha}}{2M} \right] U(P) \]

Total quark angular momentum:

\[ J^{u+d} = \frac{1}{2} \left[ A_{20}^{u+d}(0) + B_{20}^{u+d}(0) \right] = \frac{1}{2} \left[ \langle x \rangle^{u+d} + B_{20}^{u+d}(0) \right] \]

Quark angular momentum (Ji’s sum rule)

\[ J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} x dx \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] \]

GPDs And Orbital Angular Momentum Distribution:

\[ O^{\beta \mu_1 \mu_2 \ldots \mu_n} = \overline{\psi} \gamma^{(\beta} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n)} \psi \]

Define generalized angular momentum tensor:

\[ M^{\alpha \beta \mu_1 \mu_2 \ldots \mu_n} = \xi^\alpha O^{\beta \mu_1 \mu_2 \ldots \mu_n} - \xi^\beta O^{\alpha \mu_1 \mu_2 \ldots \mu_n} \text{ (minus traces)} \]

\[ \int d^4 \xi \langle p | M^{\alpha \beta \mu_1 \mu_2 \ldots \mu_n} (\xi) | p \rangle = J_n \times \text{ tensor structures} \times (2\pi)^4 \delta^4(0) \]

\[ \int d^3 \xi \langle p | M^{12\ldots+++} (\xi) | p \rangle = S^{+++} + L^{0+++} + \Delta L^{0+++} \]

\[ L(x) = \frac{1}{2} [xq(x) + xE(x) - \Delta q(x)] \]
TMD Parton Distributions

- These appear in the processes in which hadron transverse-momentum is measured, often together with TMD fragmentation functions.

- The leading-twist ones are classified by Boer, Mulders, and Tangerman (1996, 1998)
  - There are 8 of them
    - \( q(x, k_\perp) \), \( q_T(x, k_\perp) \),
    - \( \Delta q_L(x, k_\perp) \), \( \Delta q_T(x, k_\perp) \),
    - \( \delta q(x, k_\perp) \), \( \delta_L q(x, k_\perp) \),
    - \( \delta_T q(x, k_\perp) \), \( \delta_{T'} q(x, k_\perp) \)
When integrated over $p$, one gets the coordinate space density $\rho(x)=|\psi(x)|^2$

When integrated over $x$, one gets the coordinate space density $n(p)=|\psi(p)|^2$
Wigner parton distributions & offsprings (Ji)

Mother Dis. $W(r,p)$

$q(x, r_\perp, k_\perp)$

TMDPD $q(x, k_\perp)$

PDF $q(x)$

Density $\rho(r)$

Red. Wig. $q(x, r)$

X. Ji
Wigner distributions for quarks in proton

- Wigner operator (X. Ji, PRL91:062001, 2003)

\[ \hat{\mathcal{W}}_{\Gamma} (\vec{r}, k) = \int \overline{\Psi}(\vec{r} - \eta/2) \Gamma \Psi(\vec{r} + \eta/2) e^{i\vec{k} \cdot \eta} d^4 \eta, \]

- Wigner distribution: "density" for quarks having position \( \vec{r} \) and 4-momentum \( k^\mu \) (off-shell)

\[ W_{\Gamma} (\vec{r}, k) = \frac{1}{2M} \int \frac{d^3 \vec{q}}{(2\pi)^3} \left\langle \frac{\vec{q}}{2} \left| \hat{\mathcal{W}}(\vec{r}, k) \right| - \frac{\vec{q}}{2} \right\rangle \]

\[ = \frac{1}{2M} \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{r}} \left\langle \frac{\vec{q}}{2} \left| \hat{\mathcal{W}}(0, k) \right| - \frac{\vec{q}}{2} \right\rangle \]

X. Ji
Reduced Wigner Distributions and GPDs

- The 4D reduced Wigner distribution $f(r, x)$ is related to Generalized parton distributions (GPD) $H$ and $E$ through simple FT,

\[
f_{\Gamma}(\mathbf{r}, x) = \frac{1}{2M} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} e^{-i \mathbf{q} \cdot \mathbf{r}} F_{\Gamma}(x, \xi, t).
\]

\[
\frac{1}{2M} F_{\gamma^+}(x, \xi, t) = [H(x, \xi, t) - \tau E(x, \xi, t)]
\]

\[
+ i(\mathbf{s} \times \mathbf{q}) z \frac{1}{2M} [H(x, \xi, t) + E(x, \xi, t)].
\]

$t = -q^2$

$\xi \sim q_z$

H, E depend only on 3 variables. There is a rotational symmetry in the transverse plane..

X. Ji
TMD PDFs: $f_p^u(x,k_T), g_1, f_{1T}^u, h_1, h_{1L}$

GPDs: $H_p^u(x,\xi,t), E_p^u(x,\xi,t), \tilde{H}, \tilde{E}, \ldots$

Probability to find a quark $u$ in a nucleon $P$ with a certain polarization in a position $r$ and momentum $k$.

Measure momentum transfer to quark $k_T$ distributions also important for exclusive studies.

Measure momentum transfer to target. Exclusive meson data important in understanding of SIDIS measurements.

PDFs $f_p^u(x,k_T), g_1, h_1$

FFs $F_{1p}^u(t), F_{2p}^u(t)$.

Some PDFs same in exclusive and semi-inclusive analysis.

Analysis of SIDIS and DVMP are complementary.
Holography is "lensless photography" in which an image is captured not as an image focused on film, but as an interference pattern at the film. Typically, coherent light from a laser is reflected from an object and combined at the film with light from a reference beam. This recorded interference pattern actually contains much more information than a focused image, and enables the viewer to view a true three-dimensional image which exhibits parallax.
**Computed Tomography**

Computed Tomography (CT) is a powerful nondestructive evaluation (NDE) technique for producing 2-D and 3-D cross-sectional images of an object from flat X-ray images. Characteristics of the internal structure of an object such as dimensions, shape, internal defects, and density are readily available from CT images.
By varying the energy and momentum transfer to the proton we probe its interior and generate tomographic images of the proton ("femto tomography").
Impact parameter dependent PDFs

define state that is localized in $\perp$ position:

$$|p^+, \mathbf{R}_\perp = 0_\perp, \lambda\rangle \equiv N \int d^2 \mathbf{p}_\perp |p^+, \mathbf{p}_\perp, \lambda\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \frac{1}{p^+} \int dx^- d^2 x_\perp x_\perp T^{++}(x) = \sum_i x_i x_\perp = 0_\perp$ (cf.: working in CM frame in nonrel. physics)

define impact parameter dependent PDF

$$q(x, b_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{R}_\perp = 0_\perp | \bar{q}(\frac{x^-}{2}, b_\perp)\gamma^+ q(\frac{x^-}{2}, b_\perp) | p^+, \mathbf{R}_\perp = 0_\perp \rangle e^{ix^o p^+ x^-}$$
GPDs $\longleftrightarrow q(x, b_\perp)$

- nucleon-helicity nonflip GPDs can be related to distribution of partons in $\perp$ plane

$$q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i \Delta_\perp \cdot b_\perp} H(x, 0, -\Delta_\perp^2),$$

$$\Delta q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i \Delta_\perp \cdot b_\perp} \tilde{H}(x, 0, -\Delta_\perp^2),$$

- no rel. corrections to this result! (Galilean subgroup of $\perp$ boosts)

- $q(x, b_\perp)$ has probabilistic interpretation, e.g.

$$q(x, b_\perp) \geq |\Delta q(x, b_\perp)| \geq 0 \quad \text{for} \quad x > 0$$

$$q(x, b_\perp) \leq |\Delta q(x, b_\perp)| \leq 0 \quad \text{for} \quad x < 0$$

Burkardt
\[
\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x,0,-\Delta^2_\perp)
\]
\[
\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x i \Delta_y}{2M} E(x,0,-\Delta^2_\perp).
\]

Consider nucleon polarized in \(x\) direction (in IMF)
\[
|X\rangle \equiv |p^+, R_\perp = 0_\perp, \uparrow\rangle + |p^+, R_\perp = 0_\perp, \downarrow\rangle.
\]

\(\leftrightarrow\) unpolarized quark distribution for this state:

\[
q(x, b_\perp) = H(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int d^2 \Delta_\perp (2\pi)^2 E(x,0,-\Delta^2_\perp) e^{-ib_\perp \cdot \Delta_\perp}
\]

\(\bullet\) simple model: for simplicity, make ansatz where \(E_q \propto H_q\)

\[
E_u(x,0,-\Delta^2_\perp) = \frac{\kappa_u^p}{2} H_u(x,0,-\Delta^2_\perp)
\]
\[
E_d(x,0,-\Delta^2_\perp) = \kappa_d^p H_d(x,0,-\Delta^2_\perp)
\]

with \(\kappa_u^p = 2\kappa_p + \kappa_n = 1.673\) \(\kappa_d^p = 2\kappa_n + \kappa_p = -2.033\).
Burkardt

\[ \lim_{x \to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp) \]
Imaging quarks at fixed Feynman-x

- For every choice of $x$, one can use the Wigner distributions to picture the nucleon in 3-space; quantum phase-space tomography!
\[ \mathcal{O}_q^{\{\mu_1 \cdots \mu_n\}} = \bar{q} \gamma^{\{\mu_1} \overrightarrow{D} \mu_2 \cdots \overrightarrow{D} \mu_n\}} q \]

→ Generalised Form Factors

\[
\langle p', s' | \mathcal{O}^{\{\mu_1 \cdots \mu_n\}}(\Delta) | p, s \rangle = \sum_{i=0}^{n-1} \frac{1}{2} u(p', s') \gamma^{\{\mu_1} u(p, s) A_{q_n, 2i}(t) \Delta^{\mu_2} \cdots \Delta^{\mu_{2i+1}} \overline{p}^{\mu_{2i+2}} \cdots \overline{p}^{\mu_n} \\
+ \bar{u}(p', s') \gamma^{\{\mu_1} \Delta{\nu}} u(p, s) \sum_{i=0}^{n-1} B_{q_n, 2i}(t) \Delta^{\mu_2} \cdots \Delta^{\mu_{2i+1}} \overline{p}^{\mu_{2i+2}} \cdots \overline{p}^{\mu_n} \\
+ C_{q_n}(t) \frac{1}{m} \bar{u}(p', s') u(p, s) \Delta^{\mu_1} \cdots \Delta^{\mu_n} \mid_{n \text{ even}}
\]
\[ A_{10}^q (Q^2) = F_1^q (Q^2) \]
\[ B_{10}^q (Q^2) = F_2^q (Q^2) \]
\[ A_{10}^\phi (Q^2) = G_A^q (Q^2) \]
\[ B_{10}^\phi (Q^2) = G_P^q (Q^2) \]
\[ J^q = \frac{1}{2} ( A_{20}^q (0) + B_{20}^q (0) ) \]
\[ \frac{1}{2} \Sigma^q = A_{10}^\phi (0) \]
Angular Momentum $J^q = L^q + S^q = \frac{1}{2}(A^q_2 + B^q_2)$, (MSbar 4 GeV$^2$)
Generalised Form Factors, \((m_\pi \approx 950\,\text{MeV})\)
Summary of LHPC hadron structure program

- Long term program to compute all $n \leq 4$ GFF’s in dynamical lattice QCD.
- Current pion masses $m_\pi \approx 350 – 750$ MeV and lattice spacing $a \approx \frac{1}{8}$ fm.
- Status of the calculation

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<tr>
<td>$\bar{q} \Gamma_{\mu} q$</td>
<td>Done!</td>
<td>Done!</td>
<td>Almost done</td>
<td>Starting</td>
</tr>
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<td>Starting</td>
</tr>
<tr>
<td>$\bar{q} \Gamma_{(\mu} D_{\nu} D_{\rho} D_{\sigma)} q$</td>
<td>Not yet</td>
<td>Done!</td>
<td>Not yet</td>
<td>Not yet</td>
</tr>
</tbody>
</table>

- Only isovector flavor combinations for GFF’s in this round.
- Finite perturbative renormalization needed to quote results in $\overline{\text{MS}}$ scheme.

$$\langle P' S' | O_{\Gamma}^{\mu_1 \cdots \mu_n} | P S \rangle_{\overline{\text{MS}}} = Z \left( \langle P' S' | O_{\Gamma}^{\mu_1 \cdots \mu_n} | P S \rangle \right)_{\text{latt}}$$

- Lighter pion masses $m_\pi \approx 250 – 350$ MeV finished by next year.

Fleming
Nucleon $F_2/F_1$ on the Lattice (I)

PRELIMINARY

- Only $I = 1$ form factors computed so far to avoid disconnected diagrams. $F_1^{I=1} = F_{1p} - F_{1n}$ but $F_{1n}, F_{2n}$ not known accurately for $Q^2 \gtrsim 1 \text{ GeV}^2$.
- Our normalization is $F_2(Q^2) \to \kappa$ as $Q^2 \to 0$. 

Fleming
Nucleon $F_2/F_1$ on the Lattice (II)

PRELIMINARY

- $F_2^{I=1}/F_1^{I=1} \rightarrow \kappa_p - \kappa_n$ as $Q^2 \rightarrow 0$.
- PDG: $\kappa_p = 1.792847351(28)$
- PDG: $\kappa_n = -0.91304273(45)$
- So, comparison of $I = 1$ with $p-n$ could be OK with proper chiral extrapolation.

Fleming
Transverse quark distributions

\[ A_{n0}^q(-\Delta_1^2) = \int d^2 b_\perp e^{i \Delta_1 \cdot b_\perp} \int_{-1}^{1} x^{n-1} q(x, b_\perp) \]

\[ \langle b_\perp^2 \rangle_n^q = -4 \frac{A_{n0}^q(0)}{A_{n0}^q(0)} \]

\[ \lim_{x \to 1} q(x, b_\perp) \propto \delta(b_\perp^2) \]

- Higher moments \( A_{n0} \) weight \( x \sim 1 \).
- Slope of \( A_{n0}^q \) decreases as \( n \) increases.
- Slope of \( A_{10}^{u-d}(0) = -0.93(4) \) (GeV)\(^2\).
- Slope of \( A_{30}^{u-d}(0) = -0.13(3) \) (GeV)\(^2\).
- Will this continue at light pion masses?

D. Renner (LHPC/SESAM)
\[ H_q(x, \xi, t) = \int [dx][dy] \Phi_3^*(y_1, y_2, y_3) \Phi_3(x_1, x_2, x_3) T_{Hq}(x_i, y_i, x, \xi, t), \]
GPDs - Experimental Aspects

- DVCS measured at HERA (at H1 and Zeus)
- DVCS measured at JLab (fixed target, CLAS)
- DVCS planned at COMPASS, CERN
- DVMP measured at HERA
- DVMP measured at JLab
- DVMP measured (old data, 2002) at COMPASS
- DDVCS planned at JLab
Some Generalities

\[
\frac{1}{x-\xi + i\varepsilon} = P \left( \frac{1}{x-\xi} \right) - i\pi \delta(x-\xi)
\]

\[
\Rightarrow \text{Im}\{F\} = \pi \sum e_q^2 \left\{ F^q (\xi, \xi, t, Q^2) \cdot m F^q (-\xi, \xi, t, Q^2) \right\} 
\]

\[
\text{Re}\{F\} = -\sum e_q^2 P \int_{-1}^{+1} dx F^q (x, \xi, t, Q^2) \left\{ \frac{1}{x-\xi} \pm \frac{1}{x+\xi} \right\}
\]
small signal

![DVCS](image)

big noise

\[ A_{LU}(\phi) = \frac{d\sigma^+(\phi) - d\sigma^-(\phi)}{d\sigma^+(\phi) + d\sigma^-(\phi)} \]  

(Beam Spin Asymmetry, BSA)

\[ A_C(\phi) = \frac{d\sigma^+(\phi) - d\sigma^-(\phi)}{d\sigma^+(\phi) + d\sigma^-(\phi)} \]  

(Beam Charge Asymmetry, BCA)

find that:

\[ A_{LU}(\phi) \propto \text{Im}(\mathcal{M}_0) \sin \phi \quad \text{and} \quad A_C(\phi) \propto \text{Re}(\mathcal{M}_0) \cos \phi \]

where:

\[ \mathcal{M}_0 = \frac{\sqrt{t_0 - t}}{2m} \left[ F_1H + \xi(F_1 + F_2)\mathcal{M}_0 - \frac{t}{4m^2}E \right] \]
Kinematical domain

Collider:
H1 & ZEUS $0.0001 < x < 0.01$

Fixed target:
JLAB 6-11 GeV SSA, BCA?
HERMES 27 GeV SSA, BCA

COMPASS could provide data on:
Cross section (190 GeV)
BCA (100 GeV)
Wide $Q^2$ and $x_{bj}$ ranges

Limitation due to luminosity
EXP. STATUS ON PARTON DISTR.'S

GENERALIZED PARTON DISTRIBUTIONS:

\[ H^q, \tilde{H}^q, E^q, \tilde{E}^q \]  chirally-even quark GPDs
\[ H_T^q, \tilde{H}_T^q, E_T^q, \tilde{E}_T^q \]  chirally-odd quark GPDs

FORWARD PARTON DISTRIBUTIONS:

\[ q(x, Q^2) \]  quark number density distribution \((f_1^q)\)
\[ \Delta q(x, Q^2) \]  quark helicity distribution \((g_1^q)\)
\[ \delta q(x, Q^2) \]  quark transversity distribution \((h_1^q)\)
Nowak
Helicity-flip GPDs

\[ \frac{1}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' S' | F^{\mu\alpha}(\frac{\lambda}{2} n) F^{\nu\beta}(\frac{\lambda}{2} n) | P S \rangle \]
\[ = H_{Tg}(x, \xi) \bar{U}(P' S') \frac{P^{[\mu} \gamma^{\alpha]}_{\sigma} \sigma^{\nu\beta})}{M} U(P S) \]
\[ + E_{Tg}(x, \xi) \bar{U}(P' S') \frac{P^{[\mu} \gamma^{\alpha]}_{\nu} \gamma^{[\nu} \Delta^{\beta]})}{M} U(P S) + \ldots . \]
DVCS & Bethe-Heitler (BH)

indistinguishable final state

@ HERMES

\[ \tau = |\tau_{BH}|^2 + |\tau_{DVCS}|^2 \pm \frac{\tau_{DVCS} \tau_{BH}^* + \tau_{DVCS}^* \tau_{BH}}{I} \]

one measures interference of two processes
but BH is calculable in QED

DVCS is suppressed in respect to BH

@ HERMES

M.Kopytin, DIS 2005, Madison

28.04.2005
Fig. 5. Expected DVCS asymmetries $A_{UT}^{\sin(\phi - \phi_2)} \cos \phi$ and $A_{UT}^{\cos(\phi - \phi_2)} \sin \phi$ in the Regge ansatz for $b_{val} = 1$, $b_{sea} = 1$, $J_u = 0.4$ (0.2, 0.0), $J_d = 0.0$. $E = 0$ denotes zero effective contribution from the quark GPDs $E_q$. The calculations are done at the average kinematic values as listed in Tab. 1. Projected statistical errors are shown.

Fig. 6. Expected DVCS asymmetry $A_{UT}^{\sin(\phi - \phi_2)} \cos \phi$ with $b_{val} = 1$ and $b_{sea} = \infty$ (left panel) or $b_{sea} = 1$ (right panel), $J_u = 0.4$ (0.2, 0.0), $J_d = 0.0$ in the Regge ansatz at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPDs $E_q$. The projected statistical error for 8 million DIS events is shown. The systematic error is expected to not exceed the statistical one.

"Promising sensitivity!"
Hermes (2 fb^{-1})

Im$H$ Measurement in 2006 ? *

Lepton helicity asymmetry: $A_{LU}^{sin\phi} \approx \frac{C^T_{unp}}{C^{DVCS}_{unp}}$ with

\[
C^{DVCS}_{unp} = \frac{1}{(2-x_B)^2} \left\{ x_B (1-x_B) (H\bar{H} + \bar{H}\bar{H}) - x_B^2 \left( H\bar{E}^* + \bar{E}H^* + \bar{H}\bar{E}^* + \bar{E}\bar{H}^* \right) \right\}.
\]

\[
C^T_{unp} = F_1H + \frac{x_B}{2-x_B} (F_1 + F_2)\bar{H} - \frac{t}{4M^2} F_2\bar{E}.
\]

At $-t < 0.15$ GeV$^2$:

Relative contribution of GPD $H$

$\Rightarrow$ Asymmetry $A_{LU}^{sin\phi}$ mainly depending on Im$H$

Extraction of Im$H$ possible:

$\Leftarrow$ Two different bands for different GPD param.'s

$\Leftarrow$ Solid line: $1\sigma$ stat. errors

$\Leftarrow$ Dashed line: syst. extraction uncertainty added

*) Projections: V. Korotkov, W.-D. N., NPA 711, 175c, (2002)

Nowak
**Beam-Charge Asymmetry (BCA)**

\[ C(\phi) = \frac{N^+(\phi) - N^-(\phi)}{N^+(\phi) + N^-(\phi)} \propto I \propto \pm (c_0^I + \sum_{n=1}^{3} c_n^I \cos(n\phi) + \lambda \sum_{n=1}^{2} s_n^I \sin(n\phi)) \]

**HERMES PRELIMINARY** ($t_{\gamma} = 0.12$ GeV$^2$

**HERMES PRELIMINARY** (refined analysis, $t_{\gamma} = 0.12$ GeV$^2$

\[ A_c = c_0 + c_1 \cos \phi + s_1 \sin \phi \]

\[ \chi^2/\text{ndf} = 11.47/8 \]

\[ c_0 = 0.009 \pm 0.020 \text{ (stat)} \]

\[ c_1 = 0.059 \pm 0.028 \text{ (stat)} \]

\[ s_1 = 0.094 \pm 0.028 \text{ (stat)} \]

**\( A_c \) IN EXCLUSIVE BIN: EXPECTED \( \cos(\phi) \) DEPENDENCE \( \Rightarrow \text{Re} M_{\text{unp}}^{1,1} \)**

\( \sin \phi \) DUE TO POLARIZED BEAM

\( \cos(\phi) \) MOMENTS ZERO AT HIGHER MISSING MASS

Frank Ellinghaus, JLab, USA, May 2005
THE GPD H, SUMMARY AND OUTLOOK

BCA

BSA

△: HERMES prelim./published
△: CLAS, PRL, 2001 ($\times - 1$)

- **Hydrogen data** (1996-2000), analysis almost completed
  - BCA: $1 fb^{-1} e^+$ and $1 fb^{-1} e^-$
  - BSA: $1 fb^{-1} e^+$, Pol. = 40% (exp. 2006/2007 recoil data)

BCA: high sensitivity to $t$-dependence (fact./Regge) and D-term

BSA: highest sensitivity to $b_s$ parameter in profile function

Possibility to "map out" GPD $H^u$ in the final two HERA years.
What about the GDP $E$?

$A_{UT}$: unpolarized beam, transversely pol. target

Data taking with transverse hydrogen target in progress...
$\approx 6$ million on tape

$A_{UT}^{\sin(\phi-\phi_s)\cos \phi} \sim \frac{-t}{4M_p}(F_2H_1 - F_1E_1)$

$A_{UT}^{\cos(\phi-\phi_s)\sin \phi} \rightarrow \frac{-t}{4M_p}(F_2\tilde{H}_1 - \xi F_1\tilde{E}_1)$

$\approx 8$ Mio expected in total (November 2005)

Ellinghaus
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<th>Reaction</th>
<th>Obs.</th>
<th>Expt</th>
<th>Status</th>
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<tr>
<td>$H(\pm \xi, \xi, t)$</td>
<td>$e p \rightarrow e p \gamma$ (DVCS)</td>
<td>BSA</td>
<td>CLAS</td>
<td>4.2 GeV Published PRL</td>
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<td></td>
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<td>CLAS</td>
<td>4.8 GeV Preliminary</td>
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<td>CLAS</td>
<td>5.75 GeV Preliminary</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(+σ) Hall A</td>
<td>5.75 GeV Fall 04</td>
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<tr>
<td>$H(\pm \xi, \xi, t)$</td>
<td>$e p \rightarrow e p \gamma$ (DVCS)</td>
<td>TSA</td>
<td>CLAS</td>
<td>5.65 GeV Preliminary</td>
</tr>
<tr>
<td>$E(\pm \xi, \xi, t)$</td>
<td>$e (n) \rightarrow e n \gamma$ (DVCS)</td>
<td>BSA</td>
<td>Hall A</td>
<td>5.75 GeV Fall 04</td>
</tr>
<tr>
<td>$(u + d)$</td>
<td>$e d \rightarrow e d \gamma$ (DVCS)</td>
<td>BSA</td>
<td>CLAS</td>
<td>5.4 GeV under analysis</td>
</tr>
<tr>
<td>$H(</td>
<td>x</td>
<td>&lt; \xi, \xi, t)$</td>
<td>$e p \rightarrow e p e^+ e^-$ (DDVCS)</td>
<td>BSA</td>
</tr>
<tr>
<td>$\int_{\gamma \gamma} H, E (u + d)$</td>
<td>$e p \rightarrow e p \rho$</td>
<td>$\sigma_L$</td>
<td>CLAS</td>
<td>4.2 GeV Published PLB</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CLAS</td>
<td>5.75 GeV under analysis</td>
</tr>
<tr>
<td>$\int_{\gamma \gamma} H, E (2u - d)$</td>
<td>$e p \rightarrow e p \omega$</td>
<td>$(\sigma_L)$</td>
<td>CLAS</td>
<td>5.75 GeV Accepted EPJA</td>
</tr>
</tbody>
</table>

$+ \text{ other meson production channels } \pi, \eta, \Phi \text{ under analyses in the three Halls.}$

M. Garcon
Quantum numbers of final meson state select different $GPDs$

- Pseudoscalar mesons $(\pi, \eta...)$: $\tilde{H}, \tilde{E}$
- Vector mesons $(\rho, \omega, \phi...)$: $H, E$ (flavour singlet)
- $f$-meson family $(f_0, f_2, ...)$: $H, E$ (flavour non-singlet)
Exclusive $\rho$ production on transverse target

$$A_{UT} \sim \text{Im}(AB^*)$$

$A \sim 2H^u + H^d$
$B \sim 2E^u + E^d$

$A \sim H^u - H^d$
$B \sim E^u - E^d$

Asymmetry depends linearly on the GPD $E$ in Ji's sum rule.

$\rho^0$ and $\rho^+$ measurements allow separation of $E^u, E^d$. 

K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

Burkert
Exclusive $\rho^0$ production on transverse target

$$A_{UT} = -\frac{2\Delta_1(\text{Im}(AB^*))/\pi}{|A|^2(1-\xi^2) - |B|^2(\xi^2+t/4m^2) - \text{Re}(AB^*)2\xi^2}$$

$A \sim 2H^u + H^d$
$B \sim 2E^u + E^d$

$E^u, E^d$ needed for angular momentum sum rule.

K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

Burkert
Hard Exclusive $\rho^0$ Production

Measurement of the cross-section $\sigma_L$

\[ R = \frac{\sigma_L}{\sigma_T} = \frac{1}{\epsilon} \frac{r_{00}^{04}}{1 - r_{00}^{04}} \]

\[ \frac{\rho^0}{\rho^0} \]

$\gamma^*_L p \rightarrow p \rho^0$

GPD calculations in terms of $H$ & $E$

- Vanderhaeghen, Guichon & Guidal -

$q\bar{q}$-exchange

 gluon-exchange \{ mechanisms considered \}

HERMES Preliminary

NMC

E665

H1

ZEUS
Hard Exclusive $\rho^0$ Production

Transverse Target Spin Asymmetry $A_{UT}$

 sizograph of $A_{UT}(\phi, \phi_s)$

- HERMES PRELIMINARY
- $e^+ p \rightarrow e^' \rho^0 p$
- $A_{UT}^\sin (\phi, \phi_s) = 0.046 \pm 0.037$
- $x = 0.09$
- $Q^2 = 2.0$ GeV$^2$
- $t' = 0.13$ GeV$^2$

- HERMES PRELIMINARY
- $e^+ p \rightarrow e^' \rho^0 p$
- $<Q^2> = 2.0$ GeV$^2$
- $<t'> = 0.13$ GeV$^2$

- HERMES PRELIMINARY
- $e^+ p \rightarrow e^' \rho^0 p$
- $<Q^2> = 2.0$ GeV$^2$
- $<t'> = 0.13$ GeV$^2$

- HERMES PRELIMINARY
- $e^+ p \rightarrow e^' \rho^0 p$
- $0.02 < x < 0.06$
- $0.06 < x < 0.10$
- $0.10 < x < 0.40$

- $-t'$ (GeV$^2$)

- $A_{UT}(\phi, \phi_s)$

- No $\sigma_L$ separation yet!
- Indication of $t$-depend. at low $x$
- possibly doubled statistics at end 2005
- $\sigma_L/\sigma_T$ separation possible

Riccardo Fabbrì

SIR 2005
Exclusive $ep \rightarrow epp^0_L$ production

CLAS (4.3 GeV)

HERMES (27 GeV)

$W = 5.4 \text{ GeV}$

$Q^2 > 2 \text{ GeV}^2$

GPD formalism approximately describes CLAS and HERMES data $Q^2 > 2 \text{ GeV}^2$

Burkert
Deeply virtual meson production

Meson and Pomeron (or two-gluon) exchange …

<table>
<thead>
<tr>
<th></th>
<th>(\rho^0)</th>
<th>((\sigma), f_2, P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega)</td>
<td>(\pi, f_2, P)</td>
<td></td>
</tr>
<tr>
<td>(\Phi)</td>
<td>(P)</td>
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</tr>
</tbody>
</table>

… or scattering at the quark level?

Flavor sensitivity of DVMP on the proton:

| \(\rho^0\)   | 2u+d, 9g/4 |
| \(\omega\)    | 2u-d, 3g/4 |
| \(\Phi\)      | s, g       |
| \(\rho^+\)    | u-d        |

\[
\frac{d\sigma_L}{dt} \propto \frac{1}{Q^4} \left[ \frac{\alpha_s}{Q} \sum_{M} \int \int \psi_M(z) \frac{1}{x \pm \xi} \mu i e (aH + bE)(x, \xi, t)dxdz \right]^2 \propto \frac{f(\xi, t)}{Q^6}
\]

M. Garcon
Exclusive $\rho$ meson production: $ep \rightarrow ep\rho$

**CLAS (4.2 GeV)**

- $x_B = 0.31$
- Regge (JML)
- GPD (MG-MVdh)

- $x_B = 0.38$

- $x_B = 0.48$

- $x_B = 0.52$

GPD formalism (beyond leading order) describes approximately data for $x_B < 0.4$, $Q^2 > 1.5$ GeV$^2$

**CLAS (5.75 GeV)**

Analysis in progress

Two-pion invariant mass spectra

M Garcon
DDVCS-BH interference generates a beam spin asymmetry sensitive to

\[ \text{Im} T^{\text{DDVCS}} \sim H(\pm x(\xi, q'), \xi, t) + K \]

The (continuously varying) virtuality of the outgoing photon allows to “tune” the kinematical point \((x, \xi, t)\) at which the GPDs are sampled (with \(|x| < \xi\)).

M. Guidal & M. Vanderhaeghen, PRL 90
A. V. Belitsky & D. Müller, PRL 90
**DDVCS: first observation of ep → epe\(^+\)e\(^-\)**

* Positrons identified among large background of positive pions
* ep→epe\(^+\)e\(^-\) cleanly selected (mostly) through missing mass ep→epe\(^+\)X
* Φ distribution of outgoing γ\(^*\) and beam spin asymmetry extracted (integrated over γ\(^*\) virtuality)

**but…**

A problem for both experiment and theory:

* 2 electrons in the final state → antisymmetrisation was not included in calculations,
  → define domain of validity for exchange diagram.
* data analysis was performed assuming two different hypotheses
  either detected electron = scattered electron
  or detected electron belongs to lepton pair from γ\(^*\)

Hyp. 2 seems the most valid
→ quasi-real photoproduction of vector mesons

M.Garcon
GPD CHALLENGES

- Goal: map out the full dependence on $x, \xi, t, Q^2$
- Develop models consistent with known forward distributions, form factors, polynomiality constraints, positivity, ...
- More lattice moments, smaller pion masses, towards unquenched QCD, ...
- Launch a world-wide program for analyzing GPDs perhaps along the lines of CTEQ for PDFs.
- High energy, high luminosity is needed to map out GPDs in deeply virtual exclusive processes such as DDVCS (JLab with 12GeV unique).