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# Revisiting coupled Shukla-Varma and Convective Cell Mode in Classical and Quantum Plasmas

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# Observations

The dusty plasmas are observed in Comets, Orion nebula, Noctilucent clouds (observed at the Earth's polar summer mesopause). They have also been found in planetary rings of Saturn and Uranus during satellite missions and can also be produced in laboratory. The presence of dust particles in fusion devices has been known for a long time, however, their possible consequences for plasma operation and performances became hot research pursuits a few years ago.

The search for ordered many-body coulomb systems in laboratory devices has been a problem of great interest over the past few decades. As the coulomb coupling parameter increases, the system can be self-organized

from a disordered gas phase to a more ordered condensed phase.

The formation of Coulomb crystals in dusty plasmas, which widely exists in various systems, such as astrophysical systems, industrial plasma processing systems, laboratory discharge systems, etc has attracted a great deal of attention. It has been found that micron-sized dust particles suspended in a gaseous plasma background of temperature of a few electron volts can be charged upto  $10^4$  electrons due to their higher mobility by comparison with ions. These large charges of massive dust particle drastically increase the coupling constant by eight orders of magnitude, and the suspended dust cloud can be turned into ordered crystal states even at room temperature.

# Drift Waves

In inhomogeneous magnetized plasmas, low frequency modes with frequency  $\omega$  much less than the cyclotron frequency  $\Omega_c$  are considered to be the most dangerous ones for the establishment of quasi-stationary high beta plasma states, necessary for the realization of thermonuclear fusion. The main common feature of these modes is that they are characterized by  $k_{\parallel} \ll k_{\perp}$ . A vorticity is associated with the leading order perpendicular velocity perturbation and therefore these modes are also referred as vortex modes. Since it is our intention to avoid too strong effects of geometry and

boundaries, we shall restrict ourselves to the consideration to the WKB case, i.e

$$k_{\perp} \gg |\nabla \ln n_0|$$

For a homogenous plasma, the electric field satisfies the following equation

$$\frac{d^2 E}{dx^2} + k^2(x) = 0 \quad (\text{A1})$$

and has the form

$$E(x, t) = A e^{i(\phi(x) - \omega t)}$$

Where  $A$  is a constant and  $\phi(x)=kx$ . For the inhomogeneous case when the plasma density is a slowly varying function of  $x$ , we keep this form and set out to find the representation of the phase  $\phi(x)$ , sometimes referred to as the Eikonal. Then, dropping the time dependence, we have

$$\frac{d E}{d x} = i A \phi' e^{i \phi}$$

and

$$\frac{d^2 E}{d x^2} = \{ i A \phi'' - A (\phi')^2 \} e^{i \phi}$$

Substituting the above expressions in A1 give

$$i \phi'' - (\phi')^2 + k^2(x) = 0 \quad (\text{A2})$$

For plasmas in which the density varies on a sufficiently long scalength, the  $\phi''$  can be taken to be small compared To  $k^2(x)$  and therefore

$$\begin{aligned}\phi' &= \pm k(x) \\ \text{and} \\ \phi'' &= \pm k'(x)\end{aligned}$$

Then from A(2), we have

$$\begin{aligned}\phi' &= [k^2(x) \pm ik'(x)]^{1/2} \\ &\simeq \pm k(x) + \frac{ik'(x)}{2k(x)}\end{aligned}$$

The condition  $\varphi''$  much less than  $k^2$  gives the required local approximation.

# Fluid Motion Perpendicular to the Magnetic Field

- The fluid approximation in the plane perpendicular to the magnetic field is assumed due to a strong magnetization of the particles. The conditions are more restrictive in the parallel direction and are basically fulfilled when the phase velocity of the perturbation is zero or much larger than the thermal velocity.

Taking the cross of the momentum equation with unit vector  $\hat{z}$  (assuming that the magnetic field points in the  $z$  direction), we have

$$\mathbf{v}_{\perp} = \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi - \frac{c}{B_0 \Omega_{ci}} \frac{d}{dt} \nabla_{\perp} \phi$$

If we assume the  $\mathbf{E} \times \mathbf{B}$  drift to be the dominating part of the perturbed velocity, we may substitute it in the polarization drift. We may then write the perpendicular velocity as

$$\mathbf{v}_{\perp} = \mathbf{v}_E + \mathbf{v}_p + \mathbf{v}_*$$

where

$$\mathbf{v}_E = \frac{1}{B_0} (\mathbf{E} \times \hat{\mathbf{z}})$$

$$\mathbf{v}_{pj} = \frac{1}{B_0 \Omega_{cj}} \left[ \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{E} \right]$$

*and*

$$\mathbf{v}_{*j} = \frac{\hat{\mathbf{z}} \times \nabla p_j}{q_j n_j B_0}$$

# Interpretation of Drifts

- The drifts derived here are fluid drifts. They may differ from actual or guiding center drifts and these differences are sometimes because the readers. The reason for the differences is that the fluid picture averages particle velocities at a point, regardless of where the guiding centers are located, while the particle drifts are obtained by first averaging over the gyromotion, thus identifying a particle with its guiding center.

# What type of dynamics this is?

Drift waves constitute the basic ingredient of electrostatic fluid drift dynamics. Advection of the density,  $n$ , perpendicular to the magnetic field,  $B$ , results from the  $E \times B$  velocity  $v_E$ . This perpendicular  $E \times B$  dynamics interacts with the parallel electron dynamics to form the structure of the drift wave. Forces on the electrons parallel to  $B$  excite a parallel current  $J_{\parallel}$ . This takes place through the ion polarisation drift caused by the inertia delay of the response of the fluid velocity to temporal changes in  $E$ .

# Derivation of Original coupled Shukla Varma and Convective cell mode

We consider here a non-uniform dusty magnetoplasma containing immobile dust grains and the equilibrium density gradient  $dn_0/dx$  and study the properties of low frequency (by comparison with  $\Omega_{ci}$ ), long wavelength (by comparison with ion gyroradius) electrostatic waves. The external magnetic field is along the z-axis. The perpendicular components of the electron and ion fluid velocities are

$$v_{e\perp} \approx \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi - \frac{ck_B T_e}{eB_0 n_{e0}} \hat{z} \times \nabla_{\perp} n_{e1} \text{ --- (1)}$$

and

$$v_{i\perp} = \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi + \frac{ck_B T_i}{eB_0 n_{e0}} \hat{z} \times \nabla_{\perp} n_{i1} - \frac{c}{B_0 \Omega_{ci}} \frac{\partial}{\partial t} \nabla_{\perp} \phi \text{ --- (2)}$$

The electron continuity and the parallel equation of motion are given by

$$\frac{\partial n_{e1}}{\partial t} + \frac{c}{B_0} \hat{z} \times \nabla \phi \cdot \nabla n_{e0} + n_{e0} \frac{\partial v_{ez}}{\partial z} = 0 \text{ --- (3)}$$

and

$$\frac{\partial v_{ez}}{\partial t} = \frac{e}{m_e} \frac{\partial}{\partial z} \left( \phi - \frac{k_B T_e}{e n_{e0}} n_{e1} \right) \text{ --- (4)}$$

Next, we substitute the perpendicular components of electrons and ions into the charge conservation equation

$$\nabla \cdot (en_{i0}v_i - en_{e0}v_e) = 0 \quad (5)$$

by imposing the quasi-neutrality approximation ( $n_{e1} \approx n_{i1}$ ), which is valid for a dense plasma in which  $\omega_{pi} \gg \omega_{ci}$ . We obtain

$$\frac{\partial \nabla_{\perp}^2 \phi}{\partial t} + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} z \times (q_d n_{d0}) \cdot \nabla \phi - \frac{B_0 \hat{\Omega}_{ci}}{c} \frac{\partial}{\partial z} \left( v_{iz} - \frac{n_{e0}}{n_{i0}} v_{ez} \right) = 0 \quad (6)$$

Where the parallel component of the ion fluid velocity perturbation  $v_{iz}$  is given by

$$\frac{\partial v_{iz}}{\partial t} = - \frac{e}{m_i} \frac{\partial}{\partial z} \left( \phi + \frac{3 k_B T_i}{e n_{i0}} n_{i1} \right) \quad (7)$$

We now consider the two limiting cases.

**CASE I.**  $|\partial/\partial t| \gg v_{Te} |\partial/\partial z|$

Using the above limit and ignoring the ion parallel dynamics, we have from (4) and (6)

$$\frac{\partial v_{ez}}{\partial t} = \frac{e}{m_e} \frac{\partial \phi}{\partial z} \quad (8)$$

and

$$\frac{\partial \nabla_{\perp}^2 \phi}{\partial t} + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4 \pi c}{B_0} \hat{z} \times (q_d n_{d0}) \cdot \nabla \phi + \frac{n_{e0} B_0 \Omega_{ci}}{n_{i0} c} \frac{\partial v_{ez}}{\partial z} = 0 \quad (9)$$

Assuming that  $v_{ez}$  and  $\phi$  are vary sinusoidally, we obtain from Eqs. (8) and (9) the linear dispersion relation for coupled Shukla-Varma and Convective Cell mode given by

$$\omega = \omega_{sv} \pm \frac{1}{2} \left( \omega_{sv}^2 + \omega_{cc}^2 \right)^{1/2}$$

*where*

$$\omega_{sv} = - \frac{4 \pi c \Omega_{ci}^2 k_y \frac{\partial (q_d n_{d0})}{\partial x}}{B_0 k_{\perp}^2 \omega_{pi}^2} \quad (10)$$

*and*

$$\omega_{cc} = \left( \frac{n_{e0}}{n_{i0}} \right)^{1/2} (\Omega_{ce} \Omega_{ci})^{1/2} \frac{k_z}{k_{\perp}}$$

# Inclusion of Thermal Effects

We rederive the coupled Shukla-Verma and convective cell mode retaining the electron pressure. Simplifying, we get

$$\omega^3 - \omega_{SV}\omega^2 - (\omega_{cc}^2 + v_{Te}^2)\omega + (\omega_{SV}k_z^2 v_{Te}^2 - \frac{\omega_{cc}^2 k_y \kappa_{ne0}}{\Omega_{ce}}) = 0 \quad (11)$$

If we assume that the last two terms in the paranthesis are equal, then we obtain

$$\omega^2 - \omega_{SV}\omega - (\omega_{cc}^2 + v_{Te}^2) = 0 \quad (12)$$

# Quantum Plasmas

- The field of quantum plasma is gaining momentum and popularity.
- The wide ranging applications of quantum plasmas include microelectronic devices, intense laser-solid density plasma interaction experiments, and in super dense astrophysical objects (neutron stars and white dwarfs).
- The quantum effects are important when the de Broglie wavelength of the charge carriers (electrons, holes/positrons) become comparable with the dimensions of the system under consideration.

□ Quantum plasmas are characterized by high density and low temperature in contrast with the classical plasmas that have low density and high temperature.

An approximate criterion for the validity of the classical description can be obtained by appealing to the Heisenberg's uncertainty principle

$$(\Delta q)(\Delta p) \geq \hbar$$

Suppose one tries to describe the motion of gas molecules by classical mechanics. Denoting the magnitude of the mean momentum of the molecule by  $p$  and the mean separation distance between molecules by  $R$ , then one would expect the classical description to be

applicable if

$$Rp \gg \hbar$$

Equivalently, we can say

$$R \gg \lambda$$

When the above equation is satisfied, the quantum description ought to be equivalent to the motion of the wave packets describing individual particles which move independently in a quasi-classical manner. The mean separation distance can be estimated by imagining each molecule at the center of a cube of side  $R$ , these cubes filling the volume  $V$ . Then

$$R^3 N = V$$

*or*

$$R = \left( \frac{V}{N} \right)^{1/3}$$

The mean momentum  $p$  can be estimated from the known mean energy  $\varepsilon$  of the molecule in the gas at temperature  $T$ , which is given by

$$\bar{E} = -\frac{\partial}{\partial \beta} \ln Z = N\varepsilon$$

*where*

$$\varepsilon = \frac{3}{2} k_B T$$

is the mean energy per molecule.

$$\frac{p^2}{2m} \approx \varepsilon = \frac{3}{2} k_B T$$

Thus

$$p \approx \sqrt{3mk_B T}$$

and

$$\lambda \approx \frac{h}{\sqrt{3mk_B T}}$$

Hence the condition becomes

$$\left(\frac{V}{N}\right)^{1/3} \gg \frac{h}{\sqrt{3mk_B T}}$$

This shows that the classical approximation ought to be applicable if the concentration  $N/V$  of the molecules is sufficiently small or if the temperature  $T$  is sufficiently high, and if the mass of the molecules is not too small. The converse is true for the quantum systems.

# Coupled Shukla-Varma and Convective Cell mode in quantum plasmas

The set of equations required to derive the coupled Shukla-Varma and the convective cell mode in a quantum magnetoplasma are the same as those in classical plasmas with the exception of parallel electron equation of motion since electrons follow the Fermi-Dirac distribution in a quantum plasma. The pressure for the nonrelativistic degenerate gas is

$$p_e = \frac{\hbar^2}{5m_e} (3\pi^2)^{2/3} n_e^{5/3} \quad (13)$$

and hence the parallel equation of motion for electrons is

$$\frac{\partial v_{ez}}{\partial t} = \frac{e}{m_e} \frac{\partial \phi}{\partial z} - \frac{1}{m_e n_e} \frac{\partial p_e}{\partial z} + \frac{\hbar^2}{2m_e} \frac{\partial}{\partial z} \left( \frac{\nabla^2 \sqrt{n_e}}{\sqrt{n_e}} \right) \quad (14)$$

Solving the set of equations as before yield the following dispersion relation for coupled Shukla-Varma and convective cell mode in a quantum magnetoplasma

$$\begin{aligned}
& \omega^4 - \omega_{SV} \omega^3 - \left[ \omega_{cc}^2 + \frac{2}{3} k_z^2 v_{Fe}^2 \left( 1 - \frac{3 H_e^2 k^2}{8 v_{Fe}^2} \right) \right] \omega^2 \\
& + \frac{2}{3} \left[ \omega_{SV} \left( 1 - \frac{3 H_e^2 k^2}{8 v_{Fe}^2} \right) k_z^2 v_{Fe}^2 - \frac{\omega_{cc}^2 k_y \kappa_{ne0}}{\omega_{ce}} v_{Fe}^2 \right] \omega \\
& - \frac{1}{4} \omega_{cc}^2 k_z^2 \frac{H_e^2 k^2}{4} = 0
\end{aligned} \tag{15}$$

Where  $H_e = \hbar/m_e$ ,  $v_{Fe} = (K_B T_{Fe}/m_e)^{1/2}$ , and  $k^2 = (k_y)^2 + (k_z)^2$ .

In general, the quantum statistical term (i.e., the term arising from the second term on the right hand side of Eq. (14) dominates the quantum Bohm potential (i.e., the term arising from the third term on the right hand side of Eq. (14) and in such a case Eq. (15) reduces to

$$\omega^3 - \omega_{SV} \omega^2 - \left[ \omega_{cc}^2 + \frac{2}{3} k_z^2 v_{Fe}^2 \right] \omega + \frac{2}{3} \left[ \omega_{SV} \left( 1 - \frac{3}{8} \frac{H_e^2 k^2}{v_{Fe}^2} \right) k_z^2 v_{Fe}^2 - \frac{\omega_{cc}^2 k_y \mathcal{K}_{ne0}}{\omega_{ce}} v_{Fe}^2 \right] = 0 \quad (16)$$

# Conclusions

- We have revisited the original coupled Shukla Varma (SV) and convective cell (CC) mode and shown how the inclusion of electron thermal effects modify the original mode.
- It has been shown that the original coupled SV and CC mode doesn't get affected by the quantum corrections and they can only be introduced in the modified limit presented here. i.e., by incorporating the electron pressure effect in the convective cell mode.