



THERMODYNAMICS OF NONCOMMUTATIVE BLACK HOLE

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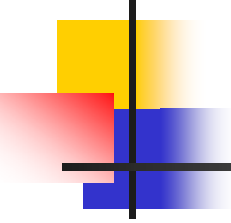
INTRODUCTION

In classical point of view black holes are such objects from which nothing, not even light can escape. But the analysis of Hawking, based on the quantum field theory revealed that black holes emit a spectrum that is analogous to a thermal black body spectrum. This shows that the thermodynamic properties of a black hole are consistent with the rest of Physics. To describe the thermodynamic properties of a black hole we have to find its temperature and hence the entropy. Usually all calculations of entropy of a black hole is based on the semi classical concept and a commutative space time.



INTRODUCTION

The entropy of a black hole is known to get corrections due to Quantum gravity, but we are interested in obtaining the modification to the entropy of black hole due to noncommutative space time. Noncommutativity is expected to be relevant at the Planck scale where it is known that usual semi classical Considerations break down. It is therefore reasonable to expect that noncommutativity would modify the entropy. If we consider the quantization of gravity then we have to consider the notion of quantized space time in which the coordinates are non-commutative. Many people investigated the noncommutative black holes.



In the most of these works solutions were not obtained directly from the Einstein's field equations. However some canonical solutions on noncommutative space were obtained directly using Einstein's field equations such as solution of noncommutative Schwarzschild black hole.

[Banerjee, Majhi and Modak Class. Quantum Grav. 26(2009)].

In this paper they incorporated the effect of noncommutativity in Mass term. The mass density is represented by Gaussian Distribution instead of Dirac delta function.

$$\rho_{\theta}(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-\frac{r^2}{4\theta}}$$



Then mass of black hole of radius r is given by

$$m_{\theta}(r) = \int_0^r 4\pi r'^2 \rho_{\theta}(r') dr' = \frac{2M}{\sqrt{\pi}} \gamma(3/2, r^2/4\theta)$$

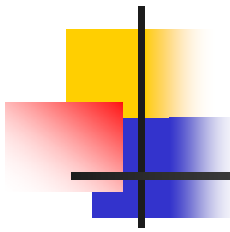
Where $\gamma(3/2, r^2/4\theta)$ is lower incomplete gamma function and given by

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt.$$

In commutative limit $\theta \rightarrow 0$, $m_{\theta}(r) \rightarrow M$ The solution of of Einstein's fields equation $(G_{\theta})^{\mu\nu} = 8\pi(T_{\theta})^{\mu\nu}$ is given by

$$ds^2 = -f_{\theta}(r)dt^2 + \frac{dr^2}{f_{\theta}(r)} + r^2 d\Omega^2 \quad (1)$$

$$f_{\theta}(r) = -(g_{\theta})_{tt} = \left(1 - \frac{4M}{r\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \right) \quad (2)$$



It is interesting to note that the noncommutative metric (1) is still stationary, static and spherically symmetric as in the commutative Case. The event horizon of the black hole can be found by setting

$$(g_{\theta})_{tt} \Big|_{r=r_h} = 0$$

in (1), we get

$$r_h = \frac{4M}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right). \quad (3)$$

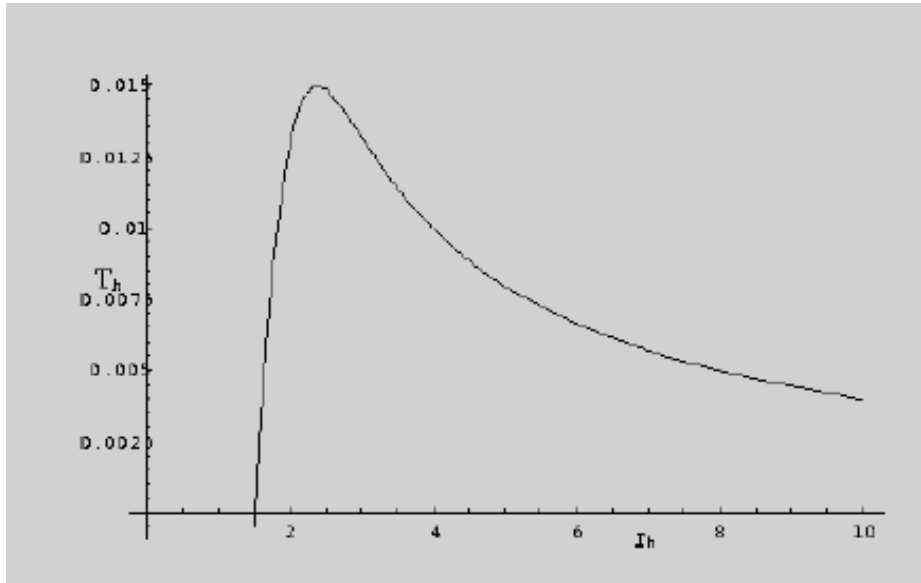
The Temperature is given by

$$T_h = \frac{\kappa}{2\pi}; \quad \kappa = \left[\frac{1}{2} \frac{d(g_{\theta})_{tt}}{dr} \right]_{r=r_h}.$$

$$T_h = \frac{1}{4\pi} \left[\frac{1}{r_h} - \frac{r_h^2}{4\theta^{3/2}} \frac{e^{-\frac{r_h^2}{4\theta}}}{\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)} \right]. \quad (4)$$



The plot of temperature verses radius is given by



Temperature is zero at $r_h = 3\sqrt{\theta}$ This is extremal limit.



According to first law of thermodynamics

$$dS_{\text{bh}} = \frac{dM}{T_h}. \quad (5)$$

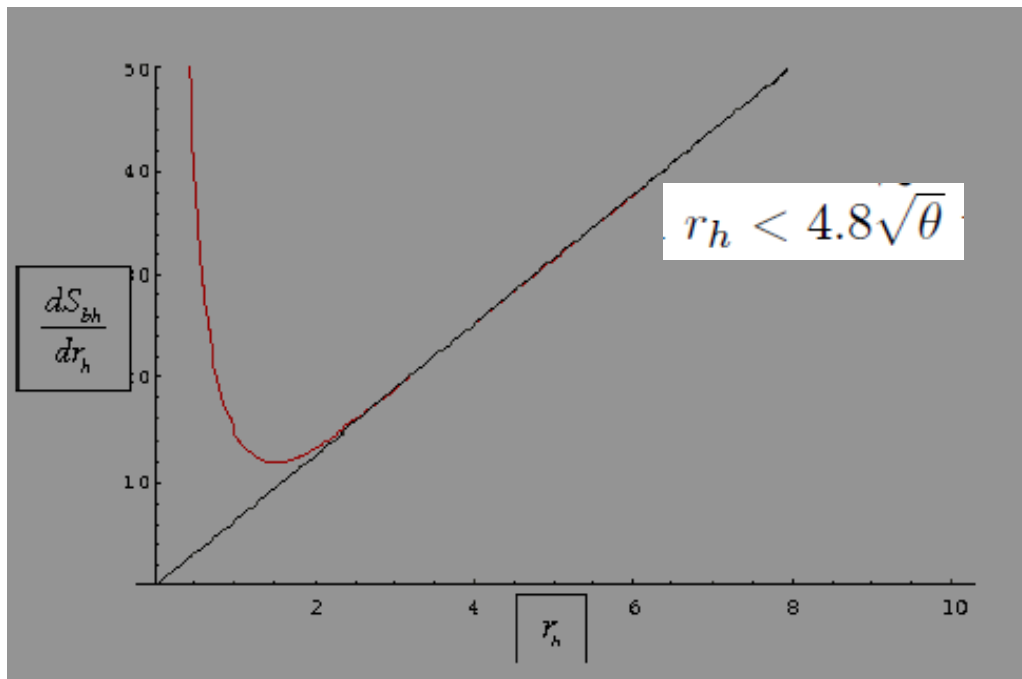
Using (3) we have

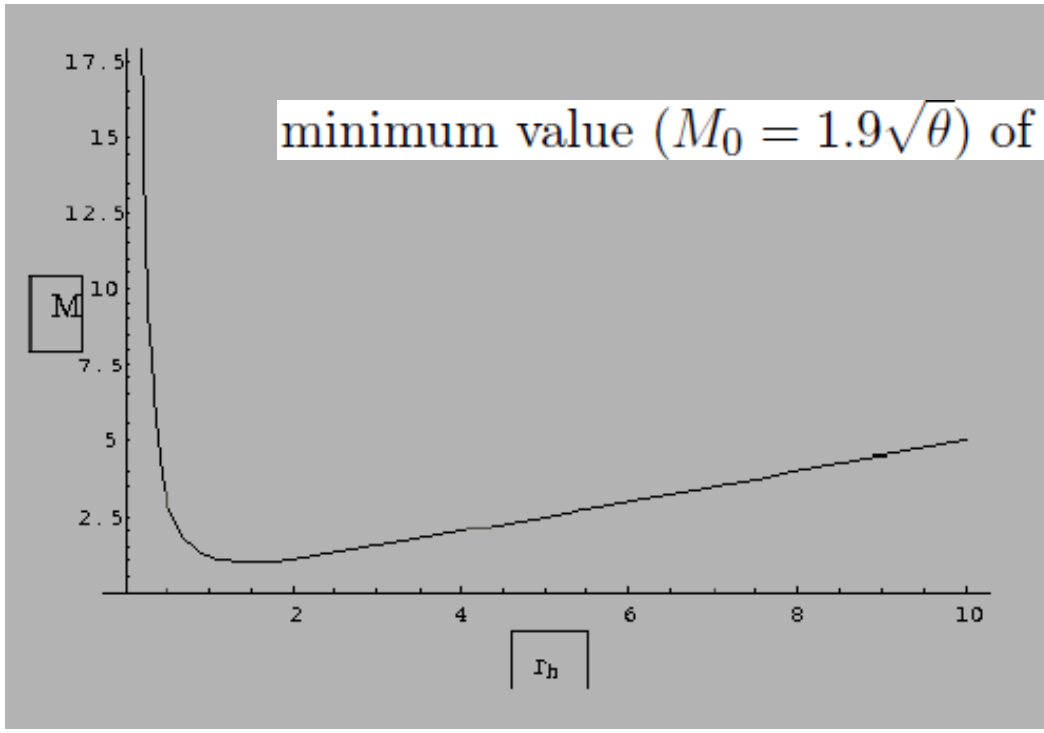
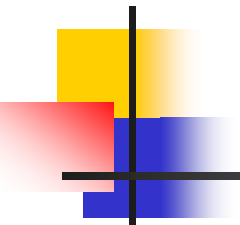
$$dM = \frac{\sqrt{\pi}}{4\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)} \left[1 - \frac{r_h^3}{4\theta^{\frac{3}{2}}} \frac{e^{-\frac{r_h^2}{4\theta}}}{\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)} \right] dr_h. \quad (6)$$

So we have using (4) and (6) in (5)

$$\frac{dS_{\text{bh}}}{dr_h} = \frac{\pi^{\frac{3}{2}} r_h}{\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)}. \quad (7)$$

The plot of $\frac{dS_{bh}}{dr_h}$ verses r_h . ($\frac{dS_{bh}}{dr_h}$ is plotted in units 4θ and r_h is plotted in units of $2\sqrt{\theta}$.)





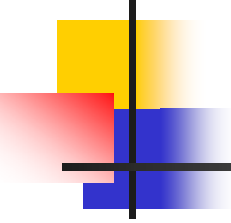
minimum value ($M_0 = 1.9\sqrt{\theta}$) of M at $r_h = 3.0\sqrt{\theta}$



Noncommutativity introduces new behavior with respect to the standard Schwarzschild black hole

- (i) Two distinct horizons occur for $M > M_0$: one inner horizon and one outer (event) horizon.
- (ii) One degenerate horizon occurs at $r_h = 3.0\sqrt{\theta}$ for $M = M_0$.
- (iii) No horizon occurs for $M < M_0$.

It is clear that from figure 1 that the semiclassical area law is not Satisfied in the $3.0\sqrt{\theta} \leq r_h < 4.8\sqrt{\theta}$ while for $r_h \gtrsim 4.8\sqrt{\theta}$ law holds. The sharp increase of $\frac{dS_{bh}}{dr_h}$ for $r_h < 3\sqrt{\theta}$ is in the unphysical domain and hence ignored.

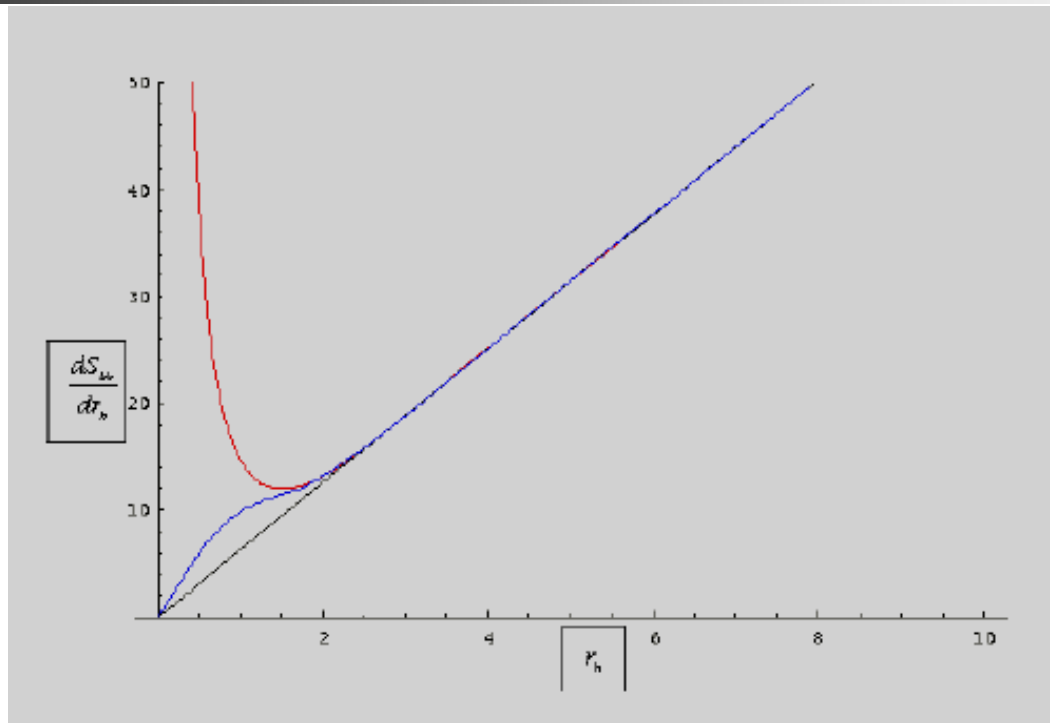


Correction to this area law such that it will describe the entropy for the complete physical region can be found as

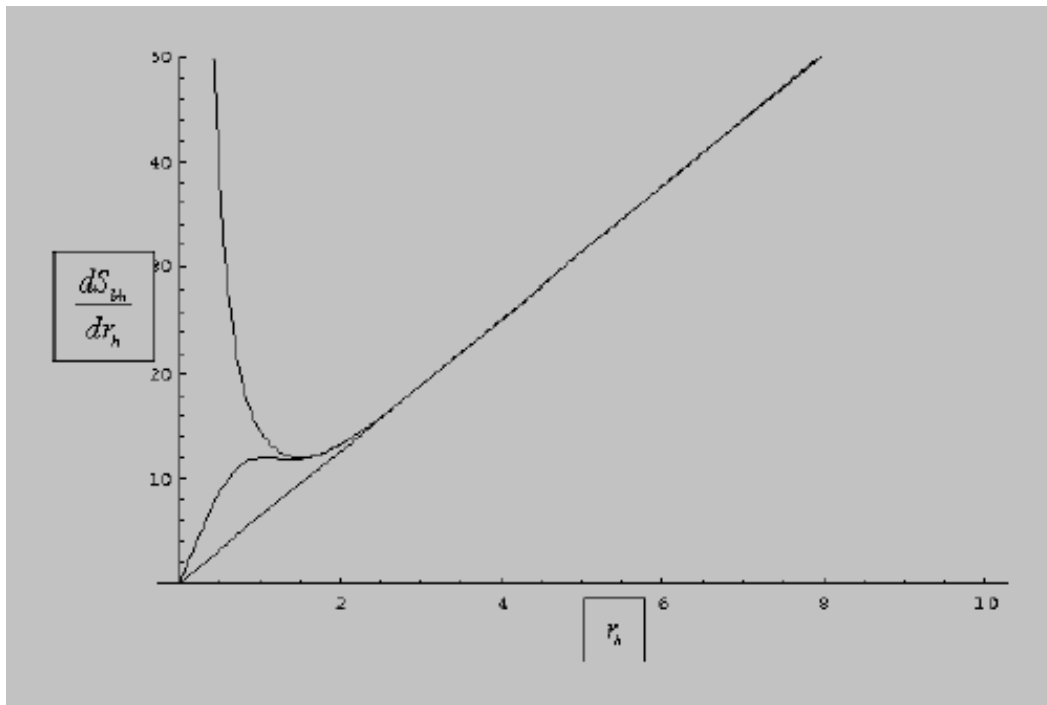
$$\frac{dS_{\text{bh}}}{dr_h} = \frac{\pi^{\frac{3}{2}} r_h}{\frac{\sqrt{\pi}}{2} - \Gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right)}$$

$$\frac{dS_{\text{bh}}}{dr_h} = 2\pi r_h \left[1 - \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) \right]^{-1}$$

$$\frac{dS_{\text{bh}}}{dr_h} = 2\pi r_h \left[1 + \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) + \frac{4}{\pi} \Gamma^2\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) + \frac{8}{\pi^{\frac{3}{2}}} \Gamma^3\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) + \dots \right].$$



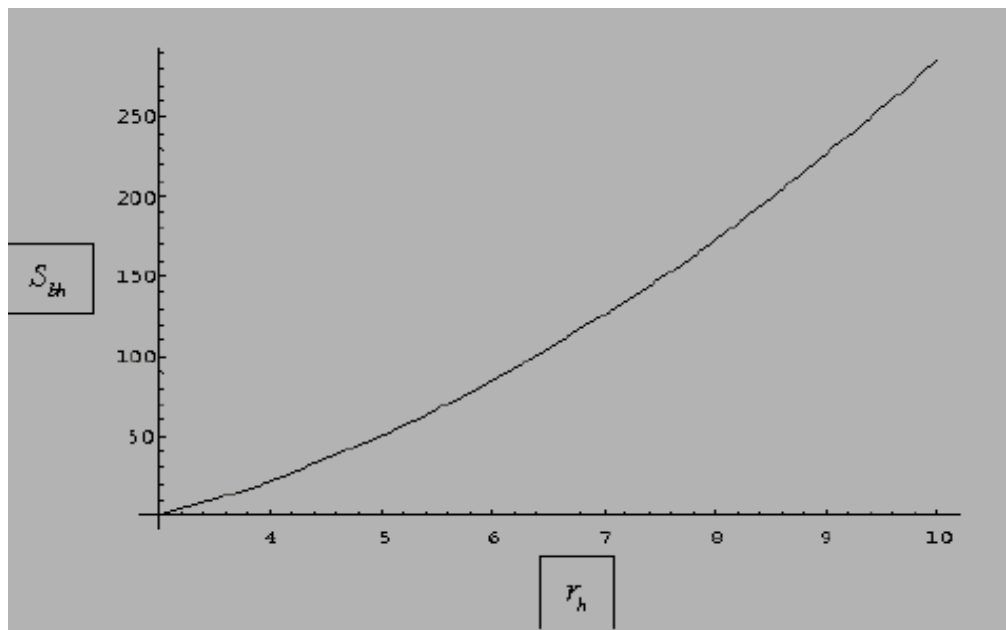
$\frac{dS_{bh}}{dr_h}$ Vs. r_h plot. $\frac{dS_{bh}}{dr_h}$ is plotted in units of 4θ and r_h is plotted in units of $2\sqrt{\theta}$



$\frac{dS_{bh}}{dr_h}$ Vs. r_h plot. $\frac{dS_{bh}}{dr_h}$ is plotted in units of 4θ and r_h is plotted in units of $2\sqrt{\theta}$

Using integration we have

$$S_{\text{bh}} = \pi r_h^2 - \sqrt{\frac{\pi}{\theta}} r_h^3 e^{-\frac{r_h^2}{4\theta}} - 6\sqrt{\pi\theta} r_h e^{-\frac{r_h^2}{4\theta}} - 6\pi\theta \left(1 - \text{Erf}\left(\frac{r_h}{2\sqrt{\theta}}\right)\right) + 2\sqrt{\pi} r_h^2 \Gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) + 8 \int r_h \Gamma^2\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) dr_h.$$



THERMODYNAMICS OF NONCOMMUTATIVE BTZ BLACK HOLE

Chang-Young (2009)
(Seiberg-witten map)

The metric of noncommutative BTZ is given by

$$ds^2 = -f^2 dt^2 + \hat{N}^{-2} dr^2 + 2r^2 N^\phi dt d\phi + \left(r^2 + \frac{\theta\beta}{2}\right) d\phi^2 + \text{order}(\theta^2)$$

$$f^2 = \frac{r^2 - r_+^2 - r_-^2}{l^2} - \frac{\theta\beta}{2l^2}, \quad N^\phi = -\frac{r_+ r_-}{r^2 l},$$

$$\hat{N}^2 = \frac{1}{l^2 r^2} [(r^2 - r_+^2)(r^2 - r_-^2) - \frac{\theta\beta}{2}(2r^2 - r_+^2 - r_-^2)]$$

θ determines the fundamental cell discretization much in the same way as the Planck constant h discretizes the phase space.



The killing horizons are given by

$$\tilde{r}_{\pm}^2 = r_{\pm}^2 \pm \frac{\beta\theta}{2} \left(\frac{r_+^2 + r_-^2}{r_+^2 - r_-^2} \right) + \text{Order}(\theta^2)$$

The apparent horizons are given by

$$r_{\pm}^2 = r_{\pm}^2 + \frac{\beta\theta}{2} + \text{Order}(\theta^2)$$

In this case inner and outer apparent(killing horizons) are not equally shifted.

Mass is given by

$$M = \frac{r^2}{(r^2 + \frac{\beta\theta}{2})^2} \left(\frac{r^2}{l^2} + \frac{J^2}{4r^2} - \frac{\beta\theta}{l^2} \right) \text{ at } r = \hat{r}_+$$

Temperature is given by

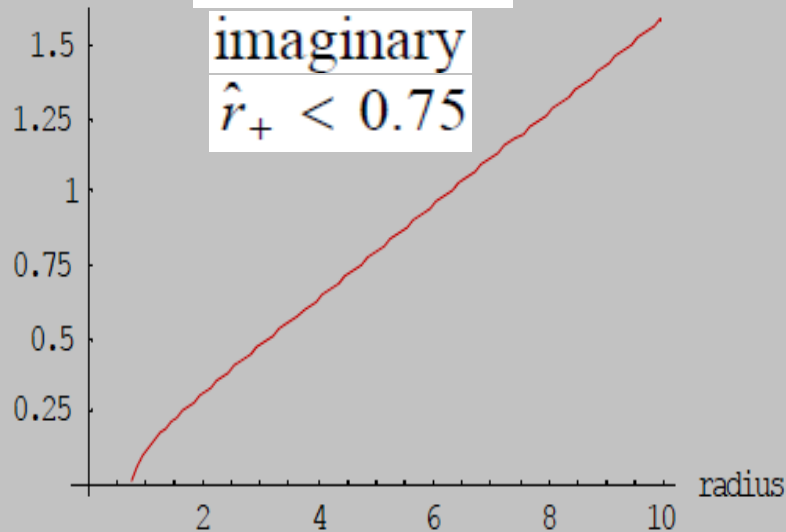
$$T_h = \frac{h}{4\pi} \sqrt{\left(\frac{2r}{l^2} - \frac{J^2 r}{2(r^2 + \frac{\beta\theta}{2})^2}\right) \left(\frac{2r}{l^2} - \frac{J^2}{2r^3} - \beta\theta \left(\frac{\frac{r^2}{(r^2 + \frac{\beta\theta}{2})^2} \left(\frac{r^2}{l^2} + \frac{J^2}{4r^2} - \frac{\beta\theta}{l^2}\right)}{r^3}\right)\right)} \quad | \quad r = \hat{r}_+$$

temperature

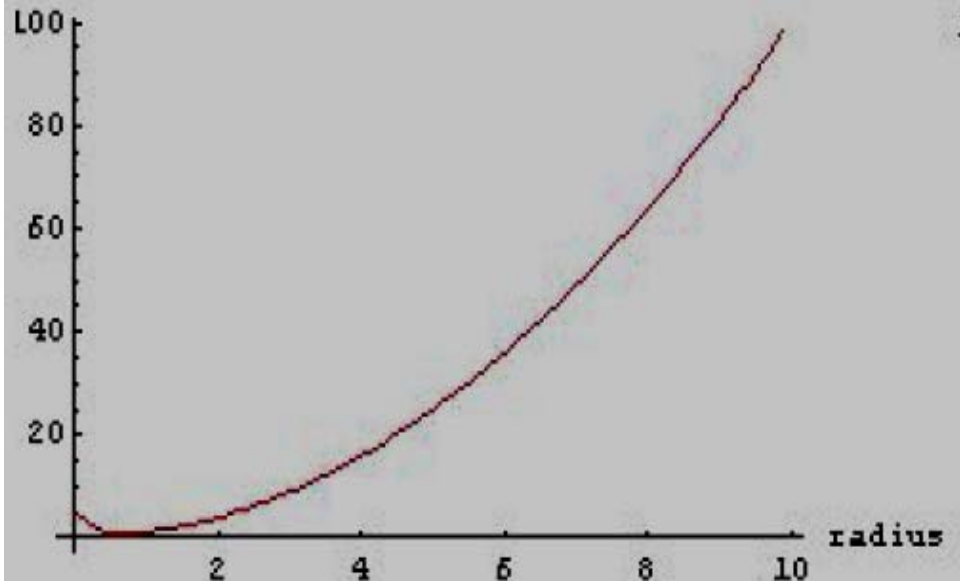
For $\theta = .1$

imaginary

$\hat{r}_+ < 0.75$



mass



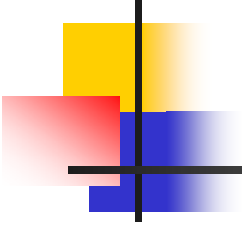


Using first law of thermodynamics

$$dS = dM/T + dJ/\Omega$$

The entropy is given by

$$dS = \frac{4\pi}{h} \left\{ \frac{-2r\left(\frac{r^4}{l^2} + \frac{J^2}{4} - \frac{\theta\beta}{l^2}r^2\right) + \left(r^2 + \frac{\theta\beta}{2}\right)\left(\frac{4r^2}{l^2} - \frac{2\theta\beta}{l^2}r\right)dr}{\left(r^2 + \frac{\theta\beta}{2}\right)^2 \sqrt{\left(\frac{2r}{l^2} - \frac{J^2r}{2\left(r^2 + \frac{\theta\beta}{2}\right)^2}\right)\left(\frac{2r}{l^2} - \frac{J^2}{2r^3} - \frac{\theta\beta}{r^2 + \frac{\theta\beta}{2}}\right)\left(\frac{r}{l^2} + \frac{J^2}{4r^3} - \frac{\theta\beta}{rl^2}\right)}} \right. \\ \left. + \frac{JdJ}{\left(r^2 + \frac{\theta\beta}{2}\right) \sqrt{\left(\frac{2r}{l^2} - \frac{J^2r}{2\left(r^2 + \frac{\theta\beta}{2}\right)^2}\right)\left(\frac{2r}{l^2} - \frac{J^2}{2r^3} - \frac{\theta\beta}{r^2 + \frac{\theta\beta}{2}}\right)\left(\frac{r}{l^2} + \frac{J^2}{4r^3} - \frac{\theta\beta}{rl^2}\right)}} \right.$$



THANK YOU ALL