Gravitational Charged Perfect Fluid Collapse in the Friedman Universe Models

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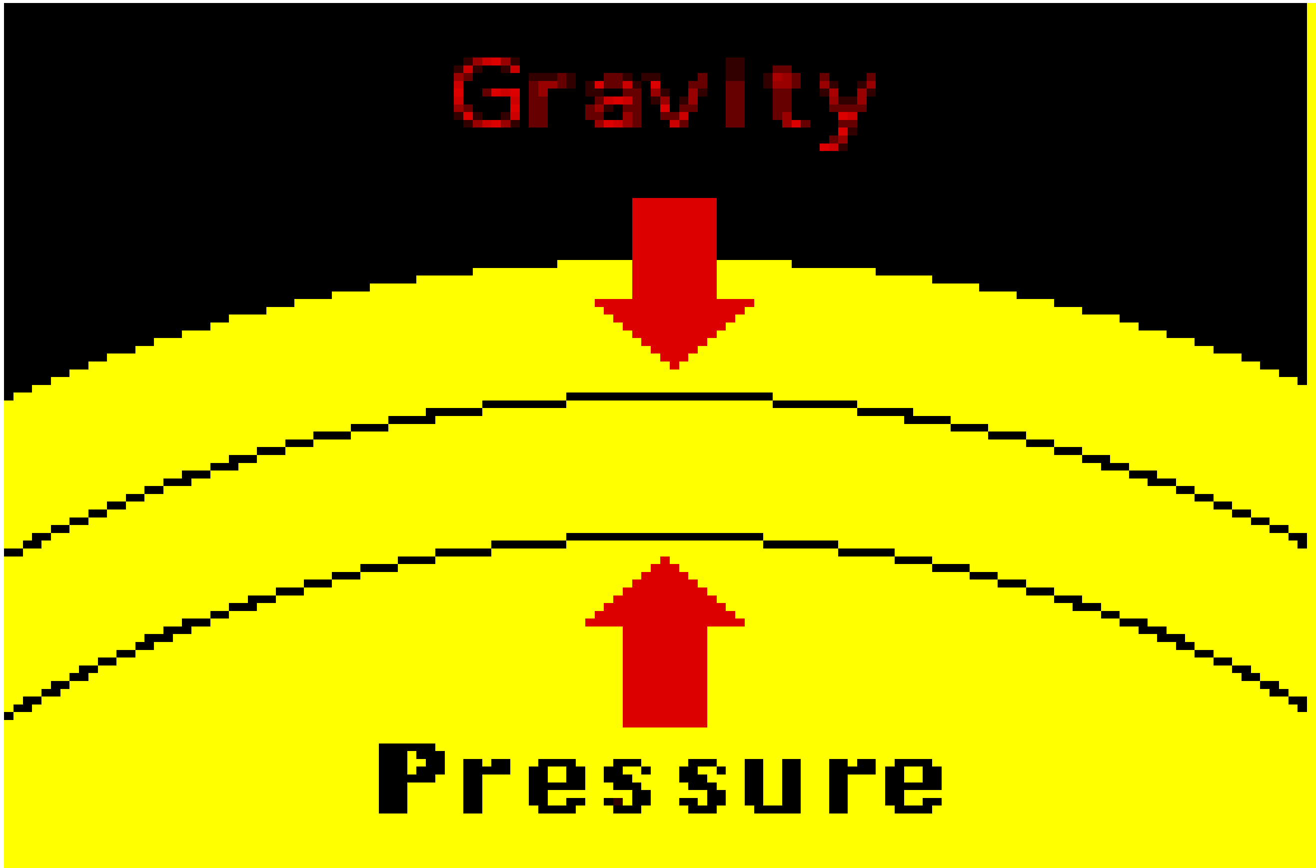
Basic Definitions

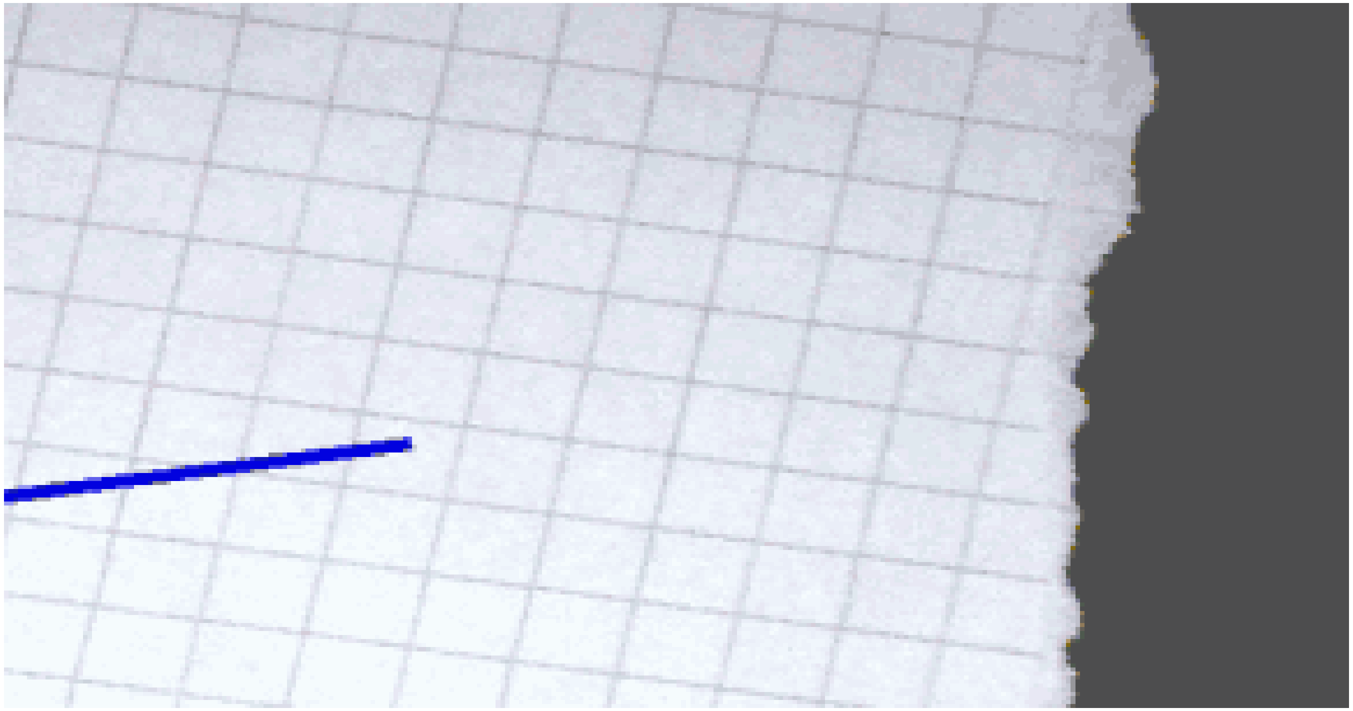
Gravitational Collapse



Basic Definitions

Spacetime Singularity

Curvature, Energy density
and pressure diverges.



Basic Definitions

Kinds of Singularities

- Curvature or essential singularity
- Coordinate Singularity

Example

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (1 - \frac{2m}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$r = 0, \quad r = 2m$

Basic Definitions

Naked Singularity

A spacetime singularity is said to be naked when it is observable to local or distant observer.

Black Hole

A spacetime singularity that can not be observed is called covered singularity or black hole.

Cosmic Censorship Hypothesis

According to this hypothesis, the singularities that appear in the gravitational collapse are always covered by an event horizon.

Literature Review

Oppenheimer and Snyder [1]
Schwarzschild solution in exterior and Friedman like solution in interior- black hole

Misner and Sharp [2]
For perfect fluid in interior.

Markovic and Shapiro [3]
Generalized the work of Oppenheimer and Snyder with positive cosmological constant.

Literature Review

Lake [4]
Both positive and negative cosmological constant.

Rocha et al. [5]
Self-similar gravitational collapse of perfect fluid using Israel's method.

Herrera and Santos [6]
Investigated the matching conditions for the collapse of perfect fluid.

Literature Review

Ghosh and Deshkar [7-8]

Collapse of radiating star with plane and spherical symmetric boundaries. Also they discuss the higher dimensional dust collapse.

Debnath et al. [9]

Non-adiabatic collapse of a quasi-spherical radiating star and discussed some thermo-dynamical relations.

Sharif and his Collaborators [10]

Darmois and Israel Junction conditions,
High speed approximation scheme.

Motivation

The cosmic censorship conjecture is a major motivation to study gravitational collapse. For this purpose, we study the gravitational collapse in the presence of electromagnetic field and cosmological constant. Main objectives of this work are

- To check the validity of CCH.
- To see the effects of electromagnetic field on the rate of collapse.

Junction Conditions

The interior metric is given by

$$ds_-^2 = dt^2 - a^2(t)d\chi^2 - a^2(t) f_k^2 (d\theta^2 + \sin^2\theta d\phi^2) \qquad (1)$$

where

$$\begin{aligned} f_k(\chi) &= \sin\chi, \quad & k &= +1 \\ &= \chi, & k &= 0 \\ &= \sinh\chi & k &= -1 \end{aligned} \qquad (2)$$

$a(t)$ is a scale factor.

Research Work

The exterior metric is taken as

$$ds_{+}^{2} = F dT^{2} - \frac{1}{F} dR^{2} - R^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}) \qquad (3)$$

where

$$F(R) = 1 - \frac{2M}{R} + \frac{Q^{2}}{R^{2}} - \frac{\Lambda R^{2}}{3}.$$

Research Work

- The continuity of line elements

$$\left(ds_{-}^{2}\right)_{\Sigma} = \left(ds_{+}^{2}\right)_{\Sigma} \qquad (4)$$

- The continuity of extrinsic curvature over $\sum$ gives

$$[k_{ab}] = k^{+}{}_{ab} - k^{-}{}_{ab} = 0 \qquad (5)$$

where the extrinsic curvature k_{ab}

$$k^{\pm}{}_{ab} = n^{\pm}{}_{\sigma}\left(\frac{\partial^{2} x \pm^{\sigma}}{\partial \xi^{a} \partial \xi^{b}} + \Gamma^{\sigma}_{\mu\nu} \frac{\partial x_{\pm}{}^{\mu} \partial x \pm^{\nu}}{\partial \xi^{a} \partial \xi^{b}}\right) \quad (a, b = 0,2,3) \qquad (6)$$

Research Work

The defining equations for hyper- surface in terms of interior and exterior co-ordinates

$$h_{-}(\chi,t)=\chi-\chi_{\Sigma}=0 \qquad (7)$$

$$h_{+}(R,T)=R-R_{\Sigma}(T)=0 \qquad (8)$$

Using Eq.(7) in Eq.(1)

$$(ds_{-}^{2})_{\Sigma}=dt^{2}-a^{2}(t)f_{k}^{2}(\chi_{\Sigma})(d\theta^{2}+\sin^{2}\theta d\phi^{2}) \qquad (9)$$

Research Work

Similarly using Eqs.(2) and (8), we get

$$(ds_+{}^2)_\Sigma = [F(R_\Sigma) - \frac{1}{F(R_\Sigma)}(\frac{dR_\Sigma}{dT})^2]dT^2 - R_\Sigma(d\theta^2 + \sin^2\theta d\varphi^2) \quad (10)$$

$$F(R_\Sigma) - \frac{1}{F(R_\Sigma)}(\frac{dR_\Sigma}{dT})^2 > 0$$

From Eqs. (4), (9) and (10), it follows that

$$R_\Sigma = a(t) f_k(\chi_\Sigma) \quad (11)$$

$$\left[F(R_\Sigma) - \frac{1}{F(R_\Sigma)}(\frac{dR_\Sigma}{dT})^2\right]^{\frac{1}{2}} dT = dt \quad (12)$$

Research Work

Now the outward normals

$$n_{\mu}^{-} = (0,\ a(t),0,0) \tag{13}$$

$$n_{\mu}^{+} = (-\frac{dR_{\Sigma}}{dt}, \frac{dT}{dt}, 0,0) \tag{14}$$

$$k_{00}^{-} = 0 \tag{15}$$

$$k_{22}^{-} = \csc^2\theta k_{33}^{-} = \left(a\, f\ f'\right)_{\Sigma} \tag{16}$$

$$k_{00}^{+} = \left(\dot{R}\ddot{T} - \ddot{R}\dot{T} - \frac{F}{2}\frac{dF}{dR}\dot{T}^{3} + \frac{3}{2F}\frac{dF}{dR}\dot{T}\dot{R}^{2}\right)_{\Sigma} \tag{17}$$

Research Work

$$k_{22}^{+} = \csc^2\theta\, k_{33}^{+} = \left(FR\dot{T}\right)_{\Sigma} \quad (18)$$

The continuity of extrinsic curvature gives

$$k_{00}^{+} = 0 \quad (19)$$

$$k_{22}^{-} = k_{22}^{+} \quad (20)$$

Using equations (16)-(20) along with Eqs.(3), (12) and (13) . The junction conditions turns out to be

$$(\dot{f}\,')_{\Sigma} = 0 \quad (21)$$

Research Work

$$M=\left(\frac{af}{2}-\frac{\Lambda}{6}a^3 f^3+\frac{Q^2}{2af}+\frac{a}{2}\dot{a}^2 f^3-\frac{a}{2} f f'^2\right)_\Sigma \qquad (22)$$

Equations (11), (12), (21) and (22) are the necessary and sufficient conditions for the smooth matching of the interior and exterior regions of a star.

Research Work

Solution of Einstein Field Equations

$$R_{\mu\nu} = \{(\rho + p)u_{\mu}u_{\nu} + \frac{1}{2}(\rho - p)g_{\mu\nu}\} + T_{\mu\nu}^{(em)}$$
$$-\frac{1}{2}g_{\mu\nu}T^{(em)} - \Lambda g_{\mu\nu} \quad (23)$$

where

$$T_{\mu\nu}^{(em)} = \frac{1}{4\pi}(-g^{\delta\omega}F_{\mu\delta}F_{\nu\omega} + \frac{1}{4}g_{\mu\nu}F_{\delta\omega}F^{\delta\omega}) \quad (24)$$

Research Work

$$F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu} \qquad (25)$$

$$F^{\mu\nu}_{;\nu} = 4\pi J^{\mu} \qquad (26)$$

$$\phi_{\mu} = (\phi(\chi, t), 0, 0, 0) \qquad (27)$$

$$J^{\mu} = \sigma u^{\mu} \qquad (28)$$

$$F_{01} = -F_{10} = -\frac{\partial \phi}{\partial \chi} \qquad (29)$$

Research Work

For $\mu=0$ in Eq. (27) and using Eq.(29), we get

$$\frac{\partial^2\phi}{\partial\chi^2}+2\frac{f'}{f}=4\pi\sigma a^2 \qquad (30)$$

Putting $\mu=1$ in (28) and using (29),

$$\frac{\partial}{\partial t}\left(\frac{1}{a^2}\frac{\partial\phi}{\partial\chi}\right)-\left(3\frac{\dot{a}}{a^3}\frac{\partial\phi}{\partial\chi}\right)=0 \qquad (31)$$

Integrating Eq. (30), we get

$$\frac{\partial\phi}{\partial\chi}=q(\chi)\frac{1}{af^2} \qquad (32)$$

Research Work

where

$$q(\chi) = 4\pi \int_0^{\chi} \sigma a^3 f^2 d\chi \qquad (33)$$

Also

$$E = \frac{q(\chi)}{Y^2}, \quad E = 4\pi \tilde{E} \qquad (34)$$

$$F_{01} = -F_{10} = -\frac{\partial \phi}{\partial \chi} = -E a \qquad (35)$$

$$T_{00}^{(em)} = \frac{1}{8\pi} E^2, \quad T_{11}^{(em)} = -\frac{1}{8\pi} E^2 a^2 \qquad (36)$$

Research Work

$$T_{22}^{(em)} = \frac{1}{8\pi} E^{2}(af)^{2}, \quad T_{33}^{(em)} = \sin^{2}\theta \, T_{22}^{(em)}$$

$$T^{(em)} = 0 \tag{37}$$

The Einstein Field Equations are

$$-3\frac{\ddot{a}}{a} = 4\pi(\rho + 3p) + E^{2} - \Lambda \tag{38}$$

$$-\frac{\ddot{a}}{a} - 2\frac{\dot{a}^{2}}{a^{2}} + 2\frac{f''}{a^{2} f} = 4\pi(p - \rho) + E^{2} + \Lambda \tag{39}$$

$$-\frac{\ddot{a}}{a} - \frac{\dot{a}^{2}}{a^{2}} + \frac{1}{a^{2}}\left[\frac{f''}{f} + \frac{f'^{2}}{f^{2}} - \frac{1}{f^{2}}\right] = 4\pi(p - \rho) - E^{2} + \Lambda \tag{40}$$

Research Work

The presence of electromagnetic field in the Freidmann model causes the distortion of its generic properties. To overcome this problem, we follow Tsagas [11] and assume that electromagnetic field is weak relative to matter , i.e., if E^2 is the electromagnetic field contribution in the system then

$$E^2 << \rho$$

Research Work

Integrating Eq.(22), we get

$$(f') = w(\chi)$$

where $w(\chi)$ is arbitrary function.

$$(\dot{a}f)^2 = W^2 - 1 + \frac{(af)^2\left(\Lambda + E^2 - 8\pi p_0\right)}{3} + \frac{2m}{af} \qquad (41)$$

Using Eqs.(41)and (22), we get

$$M = \frac{Q^2}{2af} + m(r) + [(\Lambda + E^2 - 8\pi p_0)]\frac{(af)^3}{6} \qquad (42)$$

Also

$$\widetilde{M}(t,r) = \frac{q^2}{2af} + m(r) + [\Lambda + E^2 - 8\pi p_0]\frac{(af)^3}{6} \qquad (43)$$

Research Work

Here, we assume

$$W(\chi)=1 \quad (44)$$

and the condition

$$(\Lambda + E^2 - 8\pi p_0) > 0$$

Using Eqs.(41) and (44)

$$af = \left(\frac{6m}{\Lambda + E^2 - 8\pi p_0}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}} \alpha(\chi,t) \quad (45)$$

where

$$\alpha(\chi,t) = \frac{\sqrt{3(\Lambda + E^2 - 8\pi p_0)}}{2}[t_s(\chi)-t] \quad (46)$$

Research Work

Apparent Horizons

For line element (1), the equation for apparent horizon is given by

$$g^{\alpha\beta}(af)_{,\alpha}(af)_{,\beta} = (\dot{a}f)^2 - f'^2 = 0 \qquad (47)$$

$$(\Lambda + E^2 - 8\pi p_0)(a\ f)^3 - 3af + 6m = 0 \qquad (48)$$

The positive real roots of Eq.(48) will give apparent horizons

Case (1) For $3m < \dfrac{1}{\sqrt{\Lambda + E^2 - 8\pi\ p_0}}$

Research Work

$$(af)_1 = \frac{2}{\sqrt{\Lambda + E^2 - 8\pi p_0}} \cos\frac{\psi}{3} \qquad (49)$$

$$(af)_2 = \frac{-1}{\sqrt{\Lambda + E^2 - 8\pi p_0}} (\cos\psi - \sqrt{3}\sin\frac{\psi}{3}) \qquad (50)$$

where

$$\cos\psi = -3m\sqrt{\Lambda + E^2 - 8\pi p_0} \qquad (51)$$

For $m = 0$, it follows from Eqs.(49-51) that

$$(af)_1 = \sqrt{\frac{3}{(\Lambda + E^2 - 8\pi p_0)}}, \quad (af)_2 = 0 \qquad (52)$$

Research Work

$(af)_1$ is called cosmological horizon and $(af)_2$ is called black hole horizon.

Case(2) For

$$3m = \frac{1}{\sqrt{\Lambda + E^2 - 8\pi p_0}}$$

$$(af)_1 = (af)_2 = \frac{1}{\sqrt{\Lambda + E^2 - 8\pi p_0}} = (af) \qquad (53)$$

unique horizon.

Research Work

The range for cosmological horizon and black hole horizon turns out to be

$$0 \le (af)_2 \le \frac{1}{\sqrt{(\Lambda + E^2 - 8\pi p_0)}} \le (af)_1 \le \sqrt{\frac{3}{(\Lambda + E^2 - 8\pi p_0)}} \qquad (54)$$

The black hole horizon has area

$$4\pi (af)^2 = \frac{4\pi}{\Lambda + E^2 - 8\pi p_0} \qquad (55)$$

and the cosmological horizon has area between

$$\frac{4\pi}{(\Lambda + E^2 - 8\pi p_0)} \quad \text{and} \quad \frac{12\pi}{(\Lambda + E^2 - 8\pi p_0)} \qquad (56)$$

Research Work

Case(3) For

$$3m > \frac{1}{\sqrt{(\Lambda + E^2 - 8\pi p_0)}}$$

we have no positive real roots, hence no horizon.

Eqs.(45) and (48) , we get

$$t_n = t_s - \frac{2}{\sqrt{3(\Lambda + E^2 - 8\pi p_0)}} \sinh^{-1}\left(\frac{(af)_n}{2m} - 1\right) \quad (n = 1,2) \qquad (57)$$

Research Work

From Eq.(54) it is clear that $(af)_1 \geq (af)_2$

Above inequality yields $t_1 \leq t_2$

Research Work

Formulation of Newtonian Model

The Newtonian potential is

$$\phi = \frac{1}{2}(1 - g_{00}) \qquad (58)$$

Using Eqs.(12) and (44) in Eq. (58) for exterior metric the Newtonian potential takes the form

$$\phi(R) = \frac{m}{R} + [\Lambda + E^2 - 8\pi p_0]\frac{R^2}{6} \qquad (59)$$

Research Work

The Newtonian force is

$$F = -\frac{m}{R^2} + (\Lambda + E^2 - 8\pi p_0)\frac{R}{3} \qquad (60)$$

The acceleration of the collapsing system from Eq. (41)

$$(af)^{\ddot{}} = -\frac{m}{(af)^2} + (\Lambda + E^2 - 8\pi p_0)\frac{af}{3} \qquad (61)$$

From Junction conditions $(af) = R$

$$F = (af)^{\ddot{}}$$

Research Work

For

$$R = \frac{1}{\sqrt{\Lambda + E^2 - p_0)}} \qquad m = \frac{1}{3\sqrt{\Lambda + E^2 - 8\pi p_0}}$$

F vanishes.

Conclusion

* Generalization.
* Generic properties of the model require weak electromagnetic $\Lambda > 8\pi p_0 - E^2$ field slow rate of collapse as compared to any other spacetime.
* Horizons form earlier than singularity, CCH is valid.
* Space-like singularity.
* Two horizons, instead of one.

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Thanks