Nonlinear electrostatic structures in unmagnetized pair-ion (fullerene) plasmas

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Outline of presentation

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3. Model for electrostatic waves in pair-ion plasmas
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1. Soliton

- **Soliton** is a nonlinear structure *(bell shaped)* which conserves its shape and interacts with surroundings or other solitons as an independent particle.
- Solitons are robust against perturbations.
- Soliton was first noticed in the month of August 1834 by the Scottish engineer **John Scott Russell** *(1808-1882)*, while conducting experiments to determine the most efficient design for canal boats along the Union canal linking Edinburgh with Glasgow.
- In 1885, **Korteweg and de Vries** derived a model equation *(known as KdV equation)* for the dynamics of nonlinear wave as follows,

\[
\frac{\partial u}{\partial t} + au \frac{\partial u}{\partial x} + b \frac{\partial^3 u}{\partial x^3} = 0
\]

Here \(a\) and \(b\) are non zero constants, which depends on fluid properties. The KdV equation posses a **soliton** *(or solitray wave)* solution.
Fig.1.2: Soliton solutions of stationary KdV equation. The speed of soliton depends on its amplitude.
**Soliton in Plasmas (contd.)**

**Linear waves** in plasmas are studied by assuming a harmonic wave solution \( \sim e^{i(k \cdot r - \omega t)} \) in the small amplitude limit.

When the wave amplitude is sufficiently large the nonlinearities cannot be ignored.

**Nonlinearities** in plasmas enter through

- Harmonic generation (involving fluid convection),
- Nonlinear Lorentz force,
- Ponderomotive force, trapping of the particles in the wave potential etc.

**Importance in Plasmas**

The study of soliton is important to understand the particle or energy transport mechanisms in plasmas.
When wave amplitude grows to such high values that linear perturbation theory cannot be applied to describe the interactions in plasmas. Then instead of applying Fourier analysis, we transform the original set of nonlinear equations, that describe the wave dynamics, into lowest order nonlinear equations (such as KdV equation, Burgers equation and NLS equation) whose properties are well known.

For arbitrary amplitude waves, one has to solve the nonlinear differential equations with full nonlinear glory and difficulties. This method has limitations and cannot be used in general and applicable to only some specific problems.

Reductive perturbation method is used when one cannot deal the equations with full nonlinearity and weak nonlinearity are assumed in the system. The stretched variables in space and time are defined and slow time variations are induced by the nonlinearity of the system. For example, the KdV equation is obtained for low amplitude nonlinear wave with appropriate scaling of space and time variables.
2. Brief Introduction of pair-ion Plasmas

- In usual electron-ion plasmas, asymmetry in collective phenomenon occur due to large difference of mass between electrons and ions.

- However, pair plasmas (e-p) consists of positively and negatively charged particles of same mass to keep the space, time symmetry because the mobility of equal mass particles is same in the electromagnetic field.

- Pair plasma consists of positrons and electrons have been produced in laboratory experiments. But the identification of collective modes is very difficult because the annihilation time is short compared with the plasma period.

- Therefore, attention is concentrated on the stable generation of a pair-ion plasma in laboratory consisting of positive and negative ions with an equal mass for collective modes identification.

- It has been found that fullerene (C_{60}) can be adopted as an ion source for pair-ion plasma production, based on the fact that interaction of electrons with fullerenes lead to the production of both negative and positive ions.
A fullerene is any molecule composed entirely of carbon, in the form of a hollow sphere, ellipsoid, or tube. Spherical fullerenes are also called buckyballs, and cylindrical ones are called carbon nano-tubes or buckytubes.

The discovery of fullerenes greatly expanded the number of known carbon allotropes, which until recently were limited to graphite, diamond, and amorphous carbon such as soot and charcoal.
The production process of pair-ion plasma is divided into three regions:

**Region I**: Consists of electron beam region and fullerene ion production region.

Positive ions $C_{60}^+$ are produced by electron impact ionization and low energy electrons are produced in connection with this process.

Negative ions $C_{60}^-$ are produced by attachment of these low energy electrons produced during the charging process of positive ions.

$$C_{60} + e^- \rightarrow C_{60}^+ + 2e^- \quad , \quad \sigma(C_{60}^+) = 25 \times 10^{-24} \text{ cm}^2 \quad \text{at} \ 100 \text{ eV}$$

$$C_{60} + e^- \rightarrow C_{60}^- \quad , \quad \sigma(C_{60}^-) = 100 \times 10^{-24} \text{ cm}^2 \quad \text{at} \ 5 \text{ eV}$$
Region II: Electrons and ions are rapidly separated by a magnetic filtering effect and only $C_{60}^+$ and $C_{60}^-$ are expected to exist in midmost of the cylinder.

Charge particle gyroradii $\sim (\text{mass})^{1/2}/\text{charge}$ (for same thermal velocity in the perpendicular direction of B)

Region III: Electron free pair-ion ($C_{60}^+$, $C_{60}^-$) plasma generated is attained here and plasma parameters are measured by Langmuir probes.

The typical values of pair-ion plasma density $\sim 2 \times 10^8 \text{ cm}^{-3}$ at $E_e=100\text{eV}$

The temperature of the positively and negatively charged fullerene ions lies in the range 0.3-0.5 eV.
[W. Oohara, Y. Kuwabara and R. Hatakeyama, PRE 75, 056403 (2007)]

The temperature of positive ions is found to be different from negative fullerene ions because the charging processes are quite different for both the plasma species.
W. Ohara, D. Date and H. Hatakeyama [PRL 95,175003 (2005)], firstly reported the electrostatic waves in paired fullerene-ion plasmas.

Three types of electrostatic waves can propagate parallel to the magnetic field in pair-ion plasmas i.e.,

1. Ion acoustic wave (IAW) \[ \omega^2 = C_s^2 k^2, \quad C_s^2 = \frac{\gamma T}{m} \] (acoustic speed)

2. Intermediate frequency wave (IFW) \[ \omega^2 = C_s^2 k^2 + 2\omega_p^2 \]

3. Ion plasma wave (IPW)

The IFW has the features that group velocity is negative but the phase velocity is positive i.e., the mode resembles as a backward wave.
Nonlinear Electrostatic Waves in pair-ion Plasmas

Model and nonlinear set of equations

Consider homogeneous, unmagnetized fullerene (C\textsubscript{60}) plasma consists of positively and negatively charged ions.

The positive and negative ions are considered to be isothermally heated but have different temperatures.

Using \textbf{two fluid theory}, the \textbf{Continuity equations} of positive and negative \textbf{ions} are given as

\[ \frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x} (n_+ v_+) = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x} (n_- v_-) = 0 \]  \hspace{1cm} (2)
Momentum equations for positively and negatively charged ions are described as,

\[
\frac{\partial v_+}{\partial t} + v_+ \frac{\partial v_+}{\partial x} = -\frac{e}{m_+} \frac{\partial \phi}{\partial x} - \frac{T_+}{m_+} \frac{1}{n_+} \frac{\partial n_+}{\partial x} \tag{3}
\]

\[
\frac{\partial v_-}{\partial t} + v_- \frac{\partial v_-}{\partial x} = \frac{e}{m_-} \frac{\partial \phi}{\partial x} - \frac{T_-}{m_-} \frac{1}{n_-} \frac{\partial n_-}{\partial x} \tag{4}
\]

(Where \(\phi\) is electrostatic potential) i.e., \(E = -\frac{\partial \phi}{\partial x}\)

Poisson equation gives,

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_+ - n_-) \tag{5}
\]

Equilibrium is defined as, \(n_{0+} = n_{0-} = n_0\) (say) \(\tag{6}\)

Dispersion relation

\[
\left[\frac{1}{(\omega^2 - V_{T-}^2 k^2)} + \frac{1}{(\omega^2 - V_{T+}^2 k^2)}\right] - \frac{1}{\omega_p^2} = 0 \tag{7}
\]

Where \(V_{T_a} = \left(\frac{T_a}{m}\right)^{1/2}\) and \(\omega_p = \left(\frac{4\pi n_0 e^2}{m}\right)^{1/2}\) \(m_+ = m_-(= m\) say)
Nonlinear Electrostatic Waves in pair-ion Plasmas (contd.)

Linear analysis of the modes in pair-ion plasmas

**Case (i)**  If \( T_+ = T_- \) (=T say), then Eq.(7) gives

\[
\omega^2 = 2\omega_p^2 + V_T^2 k^2
\]

which is the usual **Langmuir waves in pair-ion** unmagnetized plasmas.

**Case (ii)**  If \( T_+ \neq T_- \), then Eq.(7) gives quartic equation as follows

\[
\begin{align*}
\omega^4 - \left(V_{T_+}^2 k^2 + V_{T_-}^2 k^2 + 2\omega_p^2\right)\omega^2 + \omega_p^2 \left(V_{T_+}^2 + V_{T_-}^2\right) k^2 + k^4 V_{T_+}^2 V_{T_-}^2 &= 0 \\
\omega^2 &= \frac{1}{2} \left\{ \left(V_{T_+}^2 k^2 + V_{T_-}^2 k^2 + 2\omega_p^2\right) \pm \sqrt{\Delta} \right\} \tag{10}
\end{align*}
\]

where \( \Delta = \left(V_{T_+}^2 k^2 + V_{T_-}^2 k^2 + 2\omega_p^2\right)^2 - 4\left\{\omega_p^2 \left(V_{T_+}^2 + V_{T_-}^2\right) k^2 + k^4 V_{T_+}^2 V_{T_-}^2\right\} \)

here (+ve) and (-ve) sign gives **higher and lower frequency Langmuir waves** in pair-ion plasmas.
Nonlinear Electrostatic Waves in pair-ion Plasmas (contd.)

Under the approximation of small \( k \) (long wave length), the +ve root i.e., the **fast Langmuir wave** gives the relation,

\[
\omega^2 = 2\omega_p^2 + \frac{1}{2} (v_{T-}^2 + v_{T+}^2) k^2
\]

Under the approximation of small \( k \) (long wave length), the -ve root i.e., the **slow Langmuir wave** gives the relation,

\[
\omega^2 = \frac{1}{2} (v_{T-}^2 + v_{T+}^2) k^2
\]

which is **low frequency ion acoustic mode** in pair-ion plasmas.
Nonlinear set of equations in normalized form are written as follows:

\[
\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x} \left( n_+ v_+ \right) = 0
\]

\[
\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x} \left( n_- v_- \right) = 0
\]

\[
\frac{\partial v_+}{\partial t} + \left( v_+ \frac{\partial}{\partial x} \right) v_+ = -\frac{\partial \Phi}{\partial x} - \frac{1}{n_+} \frac{\partial n_+}{\partial x}
\]

\[
\frac{\partial v_-}{\partial t} + \left( v_- \frac{\partial}{\partial x} \right) v_- = \frac{\partial \Phi}{\partial x} - \frac{\mu}{n_-} \frac{\partial n_-}{\partial x}
\]

\[
\frac{\partial^2 \Phi}{\partial x^2} = (n_- - n_+)
\]

where \( \Phi = \left( \frac{e\phi}{T_+} \right) \) and \( \mu = \frac{T_+}{T_+} \) have been defined.

Normalization of time, space and velocity has been done by

\[
V_{T_i} = \left( \frac{T_+}{m} \right)^{\frac{1}{2}} \quad \lambda_{D_i} = \left( \frac{T_+}{4\pi n_0 e^2} \right)^{\frac{1}{2}} \quad \omega_p = \left( \frac{4\pi n_0 e^2}{m} \right)^{\frac{1}{2}}
\]
Using **reductive perturbation method**, the perturbed quantities can be expanded in the power series of $\varepsilon$ as follows:

\[ n_j = 1 + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \ldots \]

\[ v_j = \varepsilon v_j^{(1)} + \varepsilon^2 v_j^{(2)} + \ldots \]

\[ \Phi = \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \ldots \]

The stretching of independent variables are introduced in the following fashion:

\[ \xi = \varepsilon^{1/2} (x - \lambda t), \]

where $0 < \varepsilon \leq 1$ is an expansion parameter characterizing the strength of nonlinearity

\[ \tau = \varepsilon^{3/2} t \]

Now using above relations in normalized set of equations and collecting the lowest order terms of continuity and momentum equations of ions ($\sim \varepsilon^{3/2}$) and Poisson equation ($\sim \varepsilon$) gives after some simplification, i.e.,

\[ \frac{1}{(\lambda^2 - 1)} + \frac{1}{(\lambda^2 - \mu)} = 0 \]

or

\[ \lambda = \sqrt{\frac{1 + \mu}{2}} \]  \hspace{1cm} (11)
Now collecting the next higher order terms of continuity and momentum equations of ions ($\sim \varepsilon^{5/2}$) and Poisson equation ($\sim \varepsilon^2$) and using relation of parameter $\lambda$ given in Eq.(11) after some simplification, we obtain the **KdV equation** as follows:

\[
\partial_\tau \Phi^{(1)} + A \Phi^{(1)} \partial_\xi \Phi^{(1)} + B \partial^3_\xi \Phi^{(1)} = 0
\]

(12)

\[
A = \frac{\left[(3\lambda^2 - 1)(\lambda^2 - \mu)^3 - (3\lambda^2 - \mu)(\lambda^2 - 1)^3\right]}{2\lambda(\lambda^2 - 1)(\lambda^2 - \mu)[(\lambda^2 - \mu)^2 + (\lambda^2 - 1)^2]}
\]

(coefficient of nonlinear term)

\[
B = \frac{(\lambda^2 - 1)^2(\lambda^2 - \mu)^2}{2\lambda[(\lambda^2 - \mu)^2 + (\lambda^2 - 1)^2]}
\]

(coefficient of dispersive term)

**Note:** The soliton solution will be formed only when $T_+ \neq T_-$ holds.
If $T_+ \neq T_-$, then soliton solution of Eq.(12) is given by

$$\Phi^{(1)} = \phi_m \sec h^2 \left( \frac{\eta}{W} \right)$$

(13)

where

$$\phi_m = 3u_0 / A$$  \hspace{1cm} \text{(amplitude of the soliton)}$$

$$W = \sqrt{4B / u_0}$$ \hspace{1cm} \text{(width of the soliton)}$$

Transformed variable $\eta = \xi - u_0 \tau$ has been defined.

The velocity of the transformed frame $u_0$ has been normalized with thermal velocity of positive ion.
Fig.1: The electrostatic potential dips are shown for $\mu < 1$ i.e., the $T_+ > T_-$ for $\mu = 0.9$ (dotted curve), $\mu = 0.8$ (broken curve) and $\mu = 0.7$ (solid curve) at $u_0 = 0.8$. The potential dip is increased with the increase in positive ion temperature.

Fig.2: The electrostatic potential humps are shown for $\mu > 1$ i.e., the $T_+ < T_-$ for $\mu = 1.1$ (dotted curve), $\mu = 1.2$ (broken curve) and $\mu = 1.3$ (solid curve) at $u_0 = 0.8$. The potential hump is increased with the decrease in positive ion temperature.
5. Double Layers in Pair-ion Plasmas

Double Layer in Plasmas

A double layer is a structure in a plasma consists of two parallel layers with opposite electrical charge. The sheets of charge cause a strong electric field and a correspondingly sharp change in voltage (electrical potential) across the double layer. Ions and electrons, which enter the double layer are accelerated, decelerated or reflected by the electric field.

In general, double layers (which may be curved rather than flat) separate regions of plasma with quite different characteristics.

Mathematical Description of Double Layer in Pair-ion Plasmas

For double layers structures, using the stretching for independent variables as follows,

\[ \xi = \varepsilon (x - \lambda t), \quad \tau = \varepsilon^3 t \]

The perturbed quantities are expanded in the powers of \( \varepsilon \) as defined earlier.

Now collecting the higher order terms of continuity and momentum equations of ions (~\( \varepsilon^3 \)) and Poisson equation (~\( \varepsilon^4 \)) and using relation of parameter \( \lambda \) given in Eq.(11) after some simplification, we obtain the mKdV equation as follows:
The mKdV equation is described as

\[ A \partial_{\tau} \phi_1 + Q \partial_{\xi} \phi_1^2 + B \partial_{\xi} \phi_1^3 + \partial_{\xi}^3 \phi_1 = 0 \]  

(14)

where the coefficients are defined as

\[ A = 2\lambda \left[ \frac{1}{(\lambda^2 - \mu)^2} + \frac{1}{(\lambda^2 - 1)^2} \right] \]

\[ B = \left[ \frac{(3 \lambda^2 - 1)^2}{2(\lambda^2 - 1)^5} - \left( \frac{2 \lambda^2}{(\lambda^2 - 1)^4} \right)^2 + \frac{5 (3 \lambda^2 - 1)^2}{2(\lambda^2 - \mu)^5} - \left( \frac{2 \lambda^2 - \mu}{(\lambda^2 - \mu)^4} \right)^2 \right] \]

\[ Q = \frac{(3 \lambda^2 - \mu)}{2(\lambda^2 - \mu)^3} - \frac{(3 \lambda^2 - 1)}{2(\lambda^2 - 1)^3} \]

Now using the transformation \( \eta = \xi - U \tau \) the above mKdV equation
Double layer in Pair-ion Plasmas (contd.)

can be described in the form of energy integral equation as follows,
\[
\frac{1}{2} \left( \frac{\partial \phi_1}{\partial \eta} \right)^2 + V(\phi_1) = 0
\]  
(15)

where the Sagdeev potential is defined as
\[
V(\phi_1) = -\frac{UA}{2} \phi_1^2 + \frac{Q}{3} \phi_1^3 + \frac{B}{4} \phi_1^4
\]  
(16)

For the formation of DLs, the Sagdeev potential must satisfy the following conditions,
\[
V(\phi_1) = 0, \quad \frac{dV}{d\phi_1} = 0 \quad \text{at} \quad \phi_1 = 0 \quad \text{and} \quad \phi_1 = \phi_m.
\]
\[
\frac{d^2V}{d\phi_1^2} < 0 \quad \text{at} \quad \phi_1 = 0 \quad \text{and} \quad \phi_1 = \phi_m
\]  
(17)

The above conditions are satisfied only if, coefficients ‘Q’ and ‘U’ described as
\[
Q = -\frac{3}{2} B \varphi_m, \quad U = -\frac{1}{2} A \varphi_m^2
\]
Double layer in Pair-ion Plasmas (contd.)

The Sagdeev potential satisfying the DLs boundary conditions as follows,

\[ V(\varphi_1) = \frac{B}{4} \varphi_1^2 \left( \varphi_m - \varphi_1 \right)^2 \]  

(18)

Using Eq.(18), the solution of energy integral equation (15) can be written as,

\[ \varphi_1 = \frac{\varphi_m}{2} \left[ 1 - \tan h \left\{ \frac{-B}{8} \varphi_m (\eta - U \tau) \right\} \right] \]  

(19)

where amplitude and width of the DLs are defined as,

\[ \varphi_m = -\frac{2}{3} \frac{Q}{B} \quad \text{Width} = \frac{2 \sqrt{-B}}{\varphi_m} \]  

where \( B < 0 \) must hold

For \( Q > 0 \), compressive DLs and for \( Q < 0 \) rarefactive DLs are formed.

where \( \lambda = \sqrt{\frac{1+\mu}{2}} \) and \( \mu = \frac{T_-}{T_+} \) have been defined.
Double layer in Pair-ion Plasmas (contd.)

Numerical solutions of rarefactive DLs ($\mu>1$) in pair-ion plasmas

\[
\frac{T}{T_+} = 1.2 \quad \text{(solid curve)} \quad \text{and} \quad \frac{T}{T_+} = 1.4 \quad \text{(dashed curve)}
\]
Double layer in Pair-ion Plasmas (contd.)

Numerical solutions of compressive DLs ($\mu<1$) in pair-ion plasmas

$\frac{T_0}{T_+} = 0.8$ \textit{(solid curve)} and $\frac{T_0}{T_+} = 0.6$ \textit{(dashed curve)}
6. Dissipative shocks and solitons in Pair-ion Plasmas

Kadomstev-Petviashvili-Burgers (KPB) Equation in PI Plasmas

Electrostatic structures are studied in unmagnetized, weakly dissipative PI plasmas in the presence of weak transverse perturbations. The dissipation in the system is incorporated by taking into account the kinematic viscosity of both the species.

Normalized set of dynamic equations are written as

(Continuity equation for +ve ions)
\[
\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x} (n_+ u_+) + \frac{\partial}{\partial y} (n_+ v_+) = 0
\]

(Momentum equations for +ve ions)
\[
\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} + v_+ \frac{\partial u_+}{\partial y} = -\frac{\partial \varphi}{\partial x} - \frac{1}{n_+} \frac{\partial n_+}{\partial x} + \eta_+ \frac{\partial^2 u_+}{\partial x^2}
\]
\[
\frac{\partial v_+}{\partial t} + u_+ \frac{\partial v_+}{\partial x} + v_+ \frac{\partial v_+}{\partial y} = -\frac{\partial \varphi}{\partial y} - \frac{1}{n_+} \frac{\partial n_+}{\partial y} + \eta_+ \frac{\partial^2 u_+}{\partial y^2}
\]

(Continuity equation for -ve ions)
\[
\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x} (n_- u_-) + \frac{\partial}{\partial y} (n_- v_-) = 0
\]

(Momentum equations for -ve ions)
\[
\frac{\partial u_-}{\partial t} + u_- \frac{\partial u_-}{\partial x} + v_- \frac{\partial u_-}{\partial y} = -\frac{\partial \varphi}{\partial x} - \beta \frac{\partial n_-}{\partial x} + \eta_- \frac{\partial^2 u_-}{\partial x^2}
\]
\[
\frac{\partial v_-}{\partial t} + u_- \frac{\partial v_-}{\partial x} + v_- \frac{\partial v_-}{\partial y} = -\frac{\partial \varphi}{\partial y} - \beta \frac{\partial n_-}{\partial y} + \eta_- \frac{\partial^2 v_-}{\partial y^2}
\]
The Poisson equation is written as follows,

\[ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = n_- - n_+ \]

where \( \beta = T_- / T_+ \), \( \eta_+ = \frac{\mu_+}{\lambda_{D+} v_s} \) and \( \eta_- = \frac{\mu_-}{(\lambda_{D-} v_s)} \) have been defined.

In fully ionized gas, the coefficient of viscosity in the absence of magnetic field is given by

\[ \mu_{\pm} = 2.21 \times 10^{-15} \frac{T_{\pm}^{5/2} A_i^{1/2}}{Z^4 \ln \Lambda} \text{ gm/cm sec} \quad \text{Braginskii (1962)} \]

Using reductive perturbation method, the perturbed quantities can be expanded about their equilibrium values in powers of \( \varepsilon \) such that

\[
\begin{align*}
n_\alpha &= 1 + \varepsilon n_\alpha^{(1)} + \varepsilon^2 n_\alpha^{(2)} + \varepsilon^3 n_\alpha^{(3)} + \ldots \\
u_\alpha &= \varepsilon u_\alpha^{(1)} + \varepsilon^2 u_\alpha^{(2)} + \varepsilon^3 u_\alpha^{(3)} + \ldots \\
v_\alpha &= \varepsilon^{3/2} v_\alpha^{(1)} + \varepsilon^{5/2} v_\alpha^{(2)} + \varepsilon^{7/2} v_\alpha^{(3)} + \ldots \\
\varphi &= \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \varepsilon^3 \varphi^{(3)} + \ldots 
\end{align*}
\]

For weak dissipation, \( \eta_{\pm} = \varepsilon^{1/2} \eta_{0\pm} \) where \( \eta_{0\pm} \) is 0(1)
The stretching of the independent variables for 2-D electrostatic waves in PI plasmas is defined as

\[ \xi = \varepsilon^{1/2} (x - \lambda t), \quad \chi = \varepsilon y, \quad \tau = \varepsilon^{3/2} t \]

Now collecting the terms of different powers of \( \varepsilon \), we finally obtained the KPB equation as follows

\[
\frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \tau} + A \varphi \frac{\partial \varphi}{\partial \xi} + B \frac{\partial^3 \varphi}{\partial \xi^3} - C \frac{\partial^2 \varphi}{\partial \xi^2} \right) + D \frac{\partial^2 \varphi}{\partial \chi^2} = 0
\]  
(20)

where the coefficients as defined as,

\[
A = \frac{[3 \lambda^2 - 1](\lambda^2 - \beta)^3 - (3 \lambda^2 - \beta)(\lambda^2 - 1)^3]}{2 \lambda(\lambda^2 - 1)(\lambda^2 - \beta)[(\lambda^2 - 1)^2 + (\lambda^2 - \beta)^2]}
\]  
(Nonlinear coefficient)

\[
B = \frac{(\lambda^2 - 1)^2 (\lambda^2 - \beta)^2}{2 \lambda[(\lambda^2 - 1)^2 + (\lambda^2 - \beta)^2]}
\]  
(Dispersive coefficient)

\[
C = \frac{\eta_+(\lambda^2 - \beta)^2 - \eta_-(\lambda^2 - 1)^2}{2[(\lambda^2 - 1)^2 + (\lambda^2 - \beta)^2]}
\]  
(Dissipative coefficient)

\[
D = \lambda / 2
\]  
(weakly dispersive coefficient)
Dissipative shocks and solitons in Pair-ion Plasmas

Using transformation \( \zeta = k(\xi + \chi - U\tau) \), the solution of KPB equation is given by,

\[
\varphi(\zeta, \chi, \tau) = \frac{6}{25} \frac{C^2}{AB} \left[ 1 - \tan h \frac{C}{10B} \times \left\{ \zeta + \chi - \left( \frac{6C^2}{25B} + D \right)\tau \right\} \right] \\
+ \frac{3}{25} \frac{C^2}{AB} \left[ \sec h^2 \frac{C}{10B} \times \left\{ \zeta + \chi - \left( \frac{6C^2}{25B} + D \right)\tau \right\} \right]
\]  

(21)

Here \( k = \frac{C}{10B} \) (wave number of nonlinear wave) has been defined.

**Soliton solution**

The **KP equation** is obtained by putting \( C=0 \) in KPB equation, which gives soliton solution such as

\[
\varphi(\zeta, \chi, \tau) = \frac{12B}{A} \sech^2[\zeta + \chi - U\tau] 
\]

(22)

where \( U = -(4B + D) \) \( \lambda = \sqrt{\frac{1+\beta}{2}} \) and \( \beta = \frac{T_-}{T_+} \) have been defined.
Dissipative shocks and solitons in Pair-ion Plasmas

\[ \beta = \frac{T}{T_+} \]

For \( \beta = 1.4 \) (left) and \( \beta = 1.6 \) (right)

\[ T_- > T_+ \]

For \( \beta = 0.8 \) (left) and \( \beta = 0.6 \) (right)

\[ T_- > T_+ \]

\[ \eta_{0+} = 0.1 \]

\[ \eta_{0-} = \eta_{0+} (\beta)^{5/2} \]
Dissipative shocks and solitons in Pair-ion Plasmas

For $\beta=1.4$ (left) and $\beta=1.6$ (right),

$T_- > T_+$

For $\beta=0.8$ (left) and $\beta=0.6$ (right),

$T_- > T_+$
7. Conclusion

- Linear and nonlinear electrostatic waves are studied in unmagnetized pair-ion plasmas.

- Korteweg-de Vries (KdV) equation is obtained using reductive perturbation method.

- Both electrostatic potential hump and dip solitons are obtained in unmagnetized pair-ion plasmas.

- The electrostatic potential humps are formed, when the temperature of negative ion species in greater than the positive ions, while in reverse temperature conditions potential dips are obtained.

- Both compressive and rarefactive potential double layers are obtained depending on the ratio between difference in positive and negative ion species.
7. Conclusion

• Similarly, both compressive and rarefactive shocks and solitons are obtained from KPB equation depending on the temperature ratio between positive and negative ion species.

• No electrostatic structures are obtained, when the temperature of both positive and negative ions are same in unmagnetized pair-ion plasmas.
Thank You