A Search for Low Frequency Electromagnetic Waves in Unmagnetized Plasmas

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Abstract:
It is proposed that the electrostatic and thermal fluctuations are the source of magnetic fields in unmagnetized inhomogeneous plasmas. It is also pointed out that the ion acoustic wave can become electromagnetic in a nonuniform plasma under certain conditions. It is shown that the description of the so called magnetic electron drift vortex (MEDV) mode suffers from serious contradictions and it cannot exist as a pure transverse mode of an inhomogeneous electron plasma. Dispersion relation of a new low frequency electromagnetic wave is presented.
Plan of the Lecture:

1. Background of the Problem
2. Brief Introduction of Plasmas
3. Transverse Waves in Plasmas
4. LFEM Waves in Electron Plasmas
5. Ion Acoustic Wave (IAW)
6. Electromagnetic IAW
7. LFEM Ion Wave
8. Summary
1. Background of the Problem

The first experiment on laser interaction with the solid target was performed in USA [Ref. [1]: Stamper et. al. Phys. Rev. Lett. 26, 1012, 1971]. Later, many experiments were performed in USA and Russia in 1970s. Most of the results appeared in Phys. Rev. Lett., JETP [2-4]. Details can be seen in Review Article [5].

In early experiments,
Laser pulse duration $\sim 10^{-9} \text{sec}$

Intensity $\sim 10^{12} \frac{W}{cm^2}$

In many such experiments, large magnetic fields were observed,

$|B| \simeq (10^3 - 10^6) \text{Gauss}$
One of the explanations of this phenomenon was that, some unstable low frequency transverse perturbation is produced in the plasma. A pure transverse mode was theoretically discovered to explain magnetic fields generated in laser-induced plasmas. A great deal of work on linear and nonlinear studies of this mode including numerical simulations has appeared in literature during last many years. But, in our opinion, the physics of this mode is incorrect.

2. Brief Introduction of Plasmas

2.1 Definition:

Quasi-neutral statistical ensemble of charged particles which exhibits collective behavior.

★ Long range electromagnetic forces dominate over mechanical collisions of particles which are important in neutral fluids.
★ Density fluctuations of negative and positive charges create E fields and the motion produces currents and hence the B fields.
★ Debye Shielding:
  Applied electric potentials are shielded in plasmas within a distance about $\lambda_{De}$. 
\( T_e \neq 0 \), so shielding is not perfect.
\[ \lambda_{De} = \left( \frac{T_e}{4\pi n_0 e^2} \right)^{\frac{1}{2}} \rightarrow \text{Debye length} \]
Debye sphere = \( \frac{4}{3} \pi \lambda_{De}^3 \)
If there are neutrals in the plasma,
\( \nu \) - frequency of electron collisions with neutrals
\[ \omega_{pe} = \left( \frac{4\pi n_0 e^2}{m_e} \right)^{\frac{1}{2}} \rightarrow \text{electron plasma oscillation frequency and} \ L = \text{System size} \]
Then this system is a plasma, if
1. \( \nu \ll \omega_{pe} \) (E-field dominates)
2. \( \lambda_{De} \ll L \) (quasi-neutrality)
3. \( 1 \ll N_D = \left( \frac{4}{3} \pi \lambda_{De}^3 \right) n \) (statistical ensemble)
2.2 Plasma Approximation

Poisson eq.
\[ \nabla \cdot \mathbf{E} = 4\pi e(n_i - n_e) \]
Even if \( n_i \approx n_e \) is assumed, the \( \mathbf{E} \) is not zero.

Quasi neutrality means

\[ \lambda_{De}^2 k^2 << 1 \]

That is the wavelengths of perturbations are much longer than \( \lambda_{De} \). Since \( m_e \neq m_i \), we have many time scales in plasmas, So

\[ \omega_{pi} = \left( \frac{4\pi n_0 e^2}{m_i} \right)^{\frac{1}{2}} ; \omega_{pe} = \left( \frac{4\pi n_0 e^2}{m_e} \right)^{\frac{1}{2}} ; \Omega_i = \frac{eB_0}{m_i c} ; \Omega_e = \frac{eB_0}{m_e c} \]

Similarly many spatial scales exist in plasmas.

\[ \lambda_j = \frac{c}{\omega_{pi}} \text{, (collision-less skin depth of } j \text{th species, } j = e, i) \]

\[ \rho_j = \frac{v_{tj}}{\Omega_j} ; v_{tj} = \left( \frac{T_j}{m_j} \right)^{\frac{1}{2}} ; \rho_s = \frac{c_s}{\Omega_i} \text{ where } c_s = \left( \frac{T_e}{m_i} \right)^{\frac{1}{2}} \text{ and } \lambda_{De} \]
2.3 Kinetic Model:

Boltzmann eq:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \left( \frac{\partial f}{\partial t} \right)_c
\]  

(2.1)

\(f(r,v,t)\) is the distribution Function  
\(\mathbf{F}\)=force on a particle 

\(\left( \frac{\partial f}{\partial t} \right)_c \rightarrow \) Time rate of change of f due to collisions.

In hot plasmas, the collisions can be neglected. Then for jth species, we have Vlasov eq.

\[
\frac{\partial f_j}{\partial t} + \mathbf{v}_j \cdot \nabla f_j + \frac{q_j}{m_j} (\mathbf{E} + \frac{1}{c} \mathbf{v}_j \times \mathbf{B}) \cdot \frac{\partial f_j}{\partial \mathbf{v}_j} = 0
\]  

(2.2)
2.4 Fluid Model

Physical phenomena become clearer through fluid models. The simplest two-fluid equations for an ideal plasma are,

\[ m_j n_j (\partial_t + \mathbf{v}_j \cdot \nabla) \mathbf{v}_j = q_j n_j (\mathbf{E} + \frac{1}{c} \mathbf{v}_j \times \mathbf{B}) - \nabla p_j \]  
\hspace{2cm} (2.3)

\[ \partial_t n_j + \nabla \cdot (n_j \mathbf{v}_j) = 0 \]  
\hspace{2cm} (2.4)

\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \partial_t \mathbf{B} \]  
\hspace{2cm} (2.5)

\[ (\partial_t + \mathbf{v}_j \cdot \nabla) + \gamma_j p_j \nabla \cdot \mathbf{v}_j = 0 \]  
\hspace{2cm} (2.6)

\[ \nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B} \]  
\hspace{2cm} (2.7)

Here \( \gamma_j \) is the ratio of specific heats at constant pressure and volume. Plasma is a highly nonlinear complex system, in general.
Thermocuclear Fusion Reactors will be advantageous over Fission Reactors.

1. Nuclear waste problems are lesser.
2. Fuel is available for centuries in the oceans.

Coal burning:

\[
C + O_2 \rightarrow CO_2 + 4eV
\]

Fission

\[
_{92}U^{235} + 0 n^1 \rightarrow _{92}U^{236} \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1 + (200)MeV
\]

Fusion:

\[
_1D^2 + _1T^3 \rightarrow _2He^4 + 0 n^1 + 17.6MeV
\]

1kgm of \(U^{235}\) produces energy equal to \(3 \times 10^6\) kgm of coal.

In fusion, the energy release per kgm of fuel is about 4-times larger than fission.
I. Magnetic Confinement

In Tokomaks:
Many devices are studied like, pinches, mirrors and tokamaks for magnetic confinement. Tokamak is a device in which magnetic field is in the form of nested surfaces. Roughly speaking,

\[ n \sim 10^{14} \text{cm}^{-3} \]

\[ T \sim 1 \text{kev} \]

Two types of operations are possible:
(i) Pulsed operation
(ii) Steady operation

However, so far energy output is lesser than the energy input in such experiments.
II. Inertial Confinement:

Laser beams are focused on a fuel pallet, ablation takes place and the pallet is compressed to high densities and temperatures. Shock waves also help compression. Main problem is that the compression is not uniform and hence required results are not achieved. In stellar cores, the fusion takes place, but the stars remain intact due to gravity.
2.6 Earth’s Magnetosphere:

As the highly conducting solar wind interacts with the earth’s magnetic field, it compresses the field on sunward side and flows around it at supersonic speeds. The boundary is called Magnetopause. The inner region, from which the solar wind is excluded and which contains the compressed Earth’s magnetic field, is called the Magnetosphere. The particles cannot easily leave the solar wind due to frozen-in characteristic of a highly conducting plasma. Since the solar wind hits the obstacle (magnetized Earth) with supersonic speed, a Bow Shock Wave is generated where the plasma is slowed down and substantial fraction of the particle’s kinetic energy is converted into thermal energy.

The turbulent region between the shock wave (Bow Shock) and the Magnetopause is known as the Magnetosheath.

Van Allen Belts between (2-6) \( R_E \) (earth radius \( R_E = 6,371 \) km).

Solar Wind: (near Earth):

\[ n_e \sim 5 cm^{-3} \]
\[ T_e \sim 5 ev, \ T_i \sim 1 ev \]
\[ |B| \sim 5 \times 10^{-5} \text{Gauss} \]
\[ |v| \sim 3 \times 10^2 km/sec \]
Ionosphere:

The solar ultraviolet radiation ionizes a fraction of the neutral atmosphere. At about 80km, a permanent ionized population exists called Ionosphere. At high altitudes plasma sheet electrons can precipitate along magnetic field lines down to ionospheric altitudes, where they collide with and ionize neutral atmosphere particles. As a by-product, photons emitted by this process create the polar lights (Aurora). These auroras are typically observed inside the Auroral Oval.

2.7 Sun:

Age $\sim 4.5 \times 10^9$ Yrs

$M_0 \sim 1.99 \times 10^{30} kgm$

$R_0 \sim 6.96 \times 10^5 km$
★ Interior (3Regions):
  i. Core:
  \[ T \sim 1.6 \times 10^3 \text{ev} \]
  \[ n \sim 1.6 \times 10^5 \text{kgm}^{-3} \]
  It is a huge thermonuclear reactor.
  ii. Radiative Zone
  iii. Convective Zone
★ Outer Atmosphere:
  Lower region: Photosphere (visible) \[ [T \sim (0.1 - 0.4) \text{ev}]. \]
  Above photosphere is Chromosphere \[ (0.4 - 0.6) \text{ev} \]
  Corona: Beyond Chromosphere \[ T \sim \text{(few million degree)} \]. In between Chromosphere and Corona is a thin Transition Region \( \sim 500 \text{km} \) where the temperature rises about thousand times \((\sim \text{a few million degrees})\). Its an unsolved problem yet.
### 2.8 Brief Summary of Parameters of some Plasma Systems:

<table>
<thead>
<tr>
<th>Plasma System</th>
<th>$N_e (cm^{-3})$</th>
<th>$T_e (°k)$</th>
<th>H (Gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lonosphere: D∼70km</td>
<td>$10^3 cm^{-3}$</td>
<td>$2 \times 10^2$</td>
<td>0.3</td>
</tr>
<tr>
<td>E∼100km</td>
<td>$10^5$ (day) $10^3$ (night)</td>
<td>$2 - 3 \times 10^2$</td>
<td>0.3</td>
</tr>
<tr>
<td>F∼300km</td>
<td>$10^6$ (day), $10^5$ (night)</td>
<td>$10^3$</td>
<td>~0.3</td>
</tr>
<tr>
<td>Interplanetary Space</td>
<td>$1 - 10^4$</td>
<td>$102 - 103$</td>
<td>$(10^{-6} - 10^{-5})$</td>
</tr>
<tr>
<td>Solar Corona</td>
<td>$10^4 - 10^8$</td>
<td>$10^3 - 10^6$</td>
<td>$10^{-5} - 1$</td>
</tr>
<tr>
<td>Solar Chromosphere</td>
<td>$10^{12}$</td>
<td>$10^4$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>Stellar Interiors</td>
<td>$10^{27}$</td>
<td>$10^{7.5}$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>Pulsars</td>
<td>$10^{42}$ (center), $10^{12}$ (surface)</td>
<td>-</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Interstellar Space</td>
<td>$10^{-3} - 10$ (av ∼ 0.03)</td>
<td>$10^2$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Intergalactic Space</td>
<td>ν $10^{-5}$</td>
<td>$10^5 - 10^6$</td>
<td>ν $10^{-8}$</td>
</tr>
</tbody>
</table>
3. Transverse Waves in Plasmas

*Linear dispersion relation in vacuum:*

\[ \omega^2 = c^2 k^2 \]  \hspace{1cm} (3.1)

in plasmas.

\[ \omega^2 = c^2 k^2 + \omega_{pe}^2 \]  \hspace{1cm} (3.2)

Magnetized plasmas support a pure transverse low frequency wave; Alfven wave.

\[ \omega^2 = \nu_A^2 k_z^2 \]  \hspace{1cm} (3.3)

\[ \nu_A^2 = \left( \frac{B_0^2}{4\pi n_0 m_i} \right) \]
The plasma is then incompressible. This is the reason why the equation of motion could be neglected. This mode is called the Alfvén wave.
Eq. (3.3) is obtained through MHD. In case the density fluctuations are also taken into account, then (3.3) is modified using two fluid theory as,

\[ \omega^2 = \frac{v_A^2 k_z^2 (1 + \rho_s^2 k_\perp^2)}{(1 + \lambda_e^2 k_\perp^2)} \tag{3.4} \]

If \( \beta < \frac{m_e}{m_i} \), then (3.4) blueuces to

\[ \omega^2 \sim \frac{v_A^2 k_z^2}{1 + \lambda_e^2 k_\perp^2} \text{ (slow shear Alfven wave)} \tag{3.5} \]

If \( \frac{m_e}{m_i} < \beta < 1 \), (3.4) blueuces to

\[ \omega^2 = \nu_A^2 k_z^2 (1 + \rho_s^2 k_\perp^2) \text{ (Kinetic Alfven wave)} \tag{3.6} \]

\[ \beta = \frac{c_s^2}{\nu_A^2} = \frac{\text{kinetic pressure}}{\text{magnetic pressure}} = \frac{4\pi n_0 T_e}{B_0^2} \tag{3.7} \]
★ No transverse mode at ion time scale has been discovered in unmagnetized plasmas.
★ MEDV mode is believed to be a low frequency transverse-mode of inhomogeneous electron plasma. But it has been described incorrectly.
We try to look for some low frequency electromagnetic mode in nonuniform plasmas. The uniform unmagnetized plasma cannot support a low frequency electromagnetic wave.

4. Low Frequency Electromagnetic Waves in Electron Plasmas

4.1 MEDV Mode:

\[ m_e n_0 \partial_t v_{e1} = -e n_0 E_1 - \nabla p_{e1} \] (4.1)

\[ \nabla \times B_1 = \frac{4\pi}{c} J_1 \] (4.2)

\[ J_1 = -e n_0 v_{e1} \] (4.3)

\[ \nabla \times E_1 = -\frac{1}{c} \partial_t B_1 \] (4.4)
Since $p_1 = n_0 T_{e1}$ therefore energy equation becomes,

$$\frac{3}{2} n_0 \partial_t T_{e1} + \frac{3}{2} n_0 (v_{e1} \cdot \nabla) T_e = -p_0 \nabla \cdot v_{e1}$$  \hspace{1cm} (4.5)

Curl of (4.1) gives,

$$\partial_t (\nabla \times v_{e1}) = \frac{e}{m_e c} \partial_t B_1 + \frac{1}{m_e n_0} (\nabla n_0 \times \nabla T_{e1})$$  \hspace{1cm} (4.6)

Equations (4.2) and (4.3) yield,

$$v_{e1} = -\frac{c}{4\pi e n_0} \nabla \times B_1$$  \hspace{1cm} (4.7)

and hence

$$\nabla \times v_{e1} = \frac{c}{4\pi e n_0} \nabla^2 B_1$$  \hspace{1cm} (4.8)

where $\nabla n_0 \times (\nabla \times B_1) = 0$ due to the assumption $k \perp \nabla n_0 \perp B_1$. 
Equation (4.8) predicts \( \mathbf{E}_1 = E_1 \mathbf{\hat{x}} \) while \( \nabla n_0 = \mathbf{\hat{x}} \frac{dn_0}{dx} \), \( \nabla = (0, ik_y, 0) \) and \( \mathbf{B}_1 = B_1 \mathbf{\hat{z}} \) have been chosen. Note the vorticity \( \nabla \times \mathbf{v}_{e1} \) has been obtained from eq. of motion (4.6) and from Ampere’s law (4.7). Then, (4.6) and (4.8) yield,

\[
(1 + \lambda_e^2 k^2) \partial_t \mathbf{B}_1 = -\frac{c}{en_0} (\nabla n_0 \times \nabla T_{e1}) \tag{4.9}
\]

where \( \lambda_e = \frac{c}{\omega_{pe}} \). Compressibility is obtained from Ampere’s law (4.7),

\[
\nabla \cdot \mathbf{v}_{e1} = \frac{c}{4\pi n_0 e} \frac{\nabla n_0}{n_0} (\nabla \times \mathbf{B}_1) \tag{4.10}
\]

and therefore one obtains,

\[
T_{e1} = \frac{2}{3} \frac{c}{4\pi n_0 e} k_n \kappa_n B_1 \tag{4.11}
\]
Question:
What will happen if we evaluate $\nabla \cdot \mathbf{v}_{ei}$ from equation of motion (4.1)?

Now (4.9) and (4.11) give the linear dispersion relation of MEDV mode as,

$$\omega^2 = \frac{2}{3} C_0 \left( \frac{\kappa_n}{k_y} \right)^2 v_{Te}^2 k_y^2$$

(4.12)

where $C_0 = \frac{\lambda_e^2 k^2}{1 + \lambda_e^2 k^2}$ and $v_{te} = (T_e/m_e)^{\frac{1}{2}}$. The geometry of MEDV mode in cartesian co-ordinates is shown in Fig. 4.1. If $\nabla T_{e0} \neq 0$ is assumed, then (4.12) becomes,

$$\omega^2 = C_0 \frac{k_n}{k_y} \left[ \left( \frac{2}{3} \frac{\kappa_n}{k_y} - \frac{\kappa_T}{k_y} \right) \right] v_{Te}^2 k_y^2$$

(4.13)

where $\kappa_T = |\frac{1}{T_{e0}} \frac{dT_{e0}}{dx}|$ and $\nabla T_{e0} = + \frac{dT_{e0}}{dx} \hat{x}$. If $\left( \frac{2}{3} \kappa_n < \kappa_T \right)$, then the mode becomes unstable [Ref[8]: Yu & Chijin, Phys. Fluids 30, 3631(1987)]. In deriving (4.13),
\( \nabla \cdot \mathbf{E}_1 = 0, \nabla \cdot \mathbf{v}_{e1} \neq 0 \) and \( n_{e1} = 0 \) have been assumed and it does not seem to be very convincing. This wave geometry does not give a pure transverse wave [Ref.[9]: Saleem, Phys. Rev. E54, 4469 (1996)]. Let us write eq. of motion (4.1) in \( x \) and \( y \) components,

\[
\partial_t \nu_{ex1} = -\frac{e}{m_e} E_{x1} - v_{Te}^2 \left\{ \kappa_n \frac{T_{e1}}{T_0} + \kappa_T \frac{n_{e1}}{n_0} \right\} \quad (4.14)
\]

and

\[
\partial_t \nu_{ey1} = -\frac{e}{m_e} E_{y1} - v_{Te}^2 \left\{ i k_y \left( \frac{T_{e1}}{T_0} + \frac{n_{e1}}{n_0} \right) \right\} \quad (4.15)
\]

It is obvious from (4.15), that \( \nabla \cdot \mathbf{v}_{e1} \neq 0 \) but it gives \( E_{y1} = -\partial_y \varphi_1 \neq 0 \). Therefore, electric field has both the longitudinal \( (E_{1y}) \) and transverse \( (E_{1x}) \) components. Hence \( \nabla \cdot \mathbf{E} = 0 \) should not be used and it implies \( n_{e1} \neq 0 \). It is important to note that density perturbation cannot be neglected.
Then the curl of Amperes Law (4.2) yields a relation between $E_{1x}$ and $E_{1y}$ for $\omega << \omega_{pe}$ as,

$$E_{1x} = -\frac{1}{a} \frac{\kappa_n}{k_y} (iE_{1y})$$  \hspace{1cm} (4.16)

where $a = (1 + \lambda_e^2 k_y^2)$.

Using equation of motion (4.1) for $\nabla \cdot \mathbf{v}_{e1}$, instead of Ampere's law (4.10), in equation (4.5), we find,

$$W_0^2 \frac{T_{e1}}{T_0} = v_{te}^2 k_y^2 \left(1 - \frac{3}{2} \frac{\kappa_T^2}{k_y^2} - \Gamma_0^2\right) \frac{n_{e1}}{n_0} - \frac{e}{m_e} \left(ik_y E_{1y} + \frac{3}{2} \kappa_T E_{1x}\right)$$ \hspace{1cm} (4.17)

where $W_0^2 = \frac{3}{2} \omega^2 - v_{te}^2 k_y^2 \left(1 - \frac{3}{2} \frac{\kappa_T \kappa_n}{k_y^2}\right)$ and $\Gamma_0^2 = \frac{(\kappa_T - \kappa_n) \kappa_T}{k_y^2}$. 
The continuity equation yields,

\[ L_0^2 \frac{n_{e1}}{n_0} = - \left( 1 + \frac{v_{te}^2 k_y^2}{W_0^2} (1 - \kappa_n^2/k_y^2) \right) \left( i \frac{e}{m_e} k_y E_{1y} \right) \]

\[ - \left\{ 1 + \frac{3 \kappa_T}{2 \kappa_n} \frac{v_{te}^2 k_y^2}{W_0^2} (1 - \kappa_n^2/k_y^2) \right\} \left( \frac{e}{m_e} \kappa_n E_{1x} \right) \]  \hspace{1cm} (4.18)

where \( L_0^2 = \left\{ \omega^2 - v_{te}^2 k_y^2 (1 - \kappa_n^2/k_y^2) - \frac{v_{te}^4 k_y^4}{W_0^2} (1 - \kappa_n^2/k_y^2) \left( 1 - \frac{3}{2} \frac{\kappa_T^2}{k_y^2} - \Gamma_0^2 \right) \right\} \). Then Poisson eq.

\[ \nabla \cdot E_1 = -4\pi e \left( \frac{n_{e1}}{n_0} \right) \]  \hspace{1cm} (4.19)

can be written as,

\[ i k_y E_{1y} \left[ L_0^2 W_0^2 - \omega_{pe}^2 \left\{ W_0^2 + v_{te}^2 k_y^2 (1 - \kappa_n^2/k_y^2) \right\} \right] \]

\[ = (\kappa_n E_{1x}) \omega_{pe}^2 \left[ W_0^2 + \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 (1 - \kappa_n^2/k_y^2) \right] \]  \hspace{1cm} (4.20)
Equations (4.16) and (4.20) yield a linear dispersion relation in the limit \( \omega^2 \ll \omega_{pe}^2 \) as,

\[
a \left[ L_0^2 W_0^2 - \omega_{pe}^2 \left\{ W_0^2 + v_{te} k_y^2 \left( 1 - \frac{\kappa_n^2}{k_y^2} \right) \right\} \right]
\]

\[
= -\omega_{pe}^2 (\kappa_n/k_y)^2 \left[ W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 \left( 1 - \frac{\kappa_n^2}{k_y^2} \right) \right] \quad (4.21)
\]

This equation can be simplified as, [Ref.[10]: Saleem Phys. Plasmas (2009)],

\[
H_0 W^2 = -\frac{2}{3} v_{te}^2 \kappa_n^2 \left[ \left( 1 - \frac{3}{2} \frac{\kappa_T}{\kappa_n} \right) - (1 + \lambda_e^2 k_y^2) \left( 1 - \frac{3}{2} \frac{\kappa_T}{\kappa_n} \right) \right] \quad (4.22)
\]

where

\[
H_0 = \left\{ \left( 1 + \lambda_{De}^2 k_y^2 \right) + \frac{2}{3} \lambda_{De}^2 k_y^2 \right\} a - \kappa_n^2/k_y^2
\]
The quasi-neutrality is not allowed, otherwise we have \( n_{e1} = 0 \). The second term on right hand side of (4.22) will disappear if \( E_{1y} = 0 \) is assumed. The first term and the factor \( \lambda_e^2 k_y^2 \) in second term are the contributions of transverse component \( E_{1x} \). Thus one can not obtain MEDV-mode dispersion relation for \( E_{1y} = 0 \) from (4.22). It is interesting to note that equation (4.22) can be simplified to obtain,

\[
\omega^2 = \frac{2}{3 H_0} \lambda_e^2 k_y^2 (v_{te}^2 \kappa_n) \left( 1 - \frac{3}{2} \frac{\kappa_T}{\kappa_n} \right)
\]  

(4.23)

Note (4.23) contains effects of compressibility and hence the density perturbation in \( H_0 \) with the condition \( \frac{m_e}{m_i} < \lambda_{De}^2 k_y^2 \). So this electromagnetic wave has shorter wavelength compatible to electrostatic IAW which exists even for \( \lambda_{De}^2 \leq \frac{m_e}{m_i} \).
However, the instability condition for this electromagnetic mode is the same as was for MEDV mode that is

\[ \frac{2}{3} \kappa_n < \kappa_T \]  \hspace{1cm} (4.24)

The low frequency mode (4.23) is partially transverse and partially longitudinal. If \( E_{1x} = 0 \) is assumed, then equation (4.21) yields a low frequency electrostatic wave in a non-uniform unmagnetized plasma with the dispersion relation,

\[
\omega^2 = \frac{v_{te}^2 \kappa_n^2 \left( \frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right)}{\left[ (1 + \lambda_D^2 k_y^2) + \frac{2}{3} \lambda_D^2 k_y^2 \right]} \]  \hspace{1cm} (4.25)
5. IAW

Ion acoustic wave is a fundamental low frequency electrostatic mode of un-magnetized and magnetized plasmas.
It has always been treated as an electrostatic mode \( E = -\nabla \varphi \) as follows:

**5.1 Linear Dispersion Relation of IAW:**

**Ion dynamics:**

For \( B_0 = 0; \nabla n_0 = 0 \) (simple picture)

\[
m_i n_0 \partial_t v_{i1} = -e n_i \nabla \varphi_1 - \gamma_i T_i \nabla n_{i1}
\]  
(5.1)

\[
\partial_t n_{i1} + n_{i0} \nabla \cdot v_{i1} = 0
\]  
(5.2)

These two eqs. give,

\[
\left( \omega^2 - \frac{\gamma_i T_i}{m_i} k^2 \right) \frac{n_{i1}}{n_0} = \frac{T_e}{m_i} k^2 \frac{e \varphi_1}{T_e}
\]  
(5.3)

**Electron dynamics:**

\[
\omega << \nu_{te} k, \omega_{pe}
\]
So electron inertia is ignoble. Then for $m_e \to 0$,

$$0 \simeq -en_eE_1 - T_e \nabla n_e$$ \hspace{1cm} (5.4)

In linear limit (5.4) implies,

$$\frac{n_{e1}}{n_0} \simeq \frac{e\varphi_1}{T_e}$$ \hspace{1cm} (5.5)

For $n_{e1} \simeq n_{i1}$, (5.3) and (5.6) give the linear dispersion relation of electrostatic IAW,

$$\omega^2 = \nu_s^2 k^2$$ \hspace{1cm} (5.6)

where $\nu_s^2 = \left(\frac{T_e + \gamma_i T_i}{m_i}\right)$

For $T_i << T_e$, (5.6) is written as $\omega^2 = c_s^2 k^2$ where $c_s^2 = \left(\frac{T_e}{m_i}\right)$. 
6. IAW and Magnetostatic Mode

Here we shall show that transverse magnetostatic mode [Ref.[9]: Chu, Chu & Ohkawa, Phys. Rev. Lett. 41, 653 (1978)] which is obtained in the limit $\omega^2 << \omega_{pe}^2$ can couple with IAW in a nonuniform plasma [Ref.[10]: Saleem, Phys. Rev. E54, 4469 (1996)]. Let $k = k\hat{y}$, $\nabla n_0 = \hat{x}|\frac{dn_0}{dx}|$ and $E = (E_{1x}, E_{1y}, 0)$. Cold ions,

$$\partial_t v_{i1} = \frac{e}{m_i} E_1$$  \hspace{1cm} (6.1)

Continuity equation,

$$\frac{n_{i1}}{n_0} = \frac{e}{m_i \omega^2} (\kappa_n E_{1x} + i k_y E_{1y})$$  \hspace{1cm} (6.2)
We assume $\lambda_{De}^2 k_y^2 \neq 0$ and use Poisson equation which in the limit $\omega^2 << \omega_{pe}^2$ becomes,

\[
[-v_{te}^2 k_y^2 \omega^2 - \omega_{pi}^2 (\omega^2 - v_{te}^2 k_y^2) - \omega_{pe}^2 \omega^2] \, i k_y E_{1y} \approx [\omega_{pi}^2 (\omega^2 - v_{te}^2 k_y^2) + \omega_{pe}^2 \omega^2] \kappa_n E_{1x}
\]

Then curl of Amperes Law (4.16)

\[
E_{1x} = -\frac{1}{a k_y} \kappa_n (i E_{1y})
\]

and (6.3) give a linear dispersion relation as,

\[
\omega^2 = \frac{c_s^2 k_y^2 (a - \frac{\kappa_n^2}{k_y^2})}{(ab - \frac{\kappa_n^2}{k_y^2})}
\]

where $c_s^2 = \frac{T_e}{m_i}$, and $b = (1 + \lambda_{De}^2 k_y^2)$. 

Note $\lambda_{De}^2 < \lambda_e^2$ and if quasi neutrality is used due to Ampere’s law, then (6.4) yields the basic electrostatic IAW dispersion relation $\omega^2 = c_s^2 k_y^2$. If the displacement current is retained and Poisson equation is used without using $\omega^2, \omega_{pi}^2 << \omega_{pe}^2$, then one obtains a dispersion relation of coupled three waves; ion acoustic wave, electron plasma wave and high frequency transverse wave [Ref.[11]: Saleem, Watanabe & Sato, Phys. Rev. E62, 1155 (2000)].

7. Low Frequency Electromagnetic Ion Wave

Now we present a simple but interesting theoretical model for low frequency electromagnetic waves assuming ions to be cold. The electrostatic waves will also be considered and it will be shown that magnetic field perturbation is coupled with the dominant electrostatic field.

For inertialess electrons and $E = -\nabla \varphi$, the fundamental low frequency mode is ion acoustic wave,

$$\omega_s^2 = c_s^2 k_y^2$$  \hspace{1cm} (7.2)
in the quasi-neutrality limit. If dispersion effects are included, one obtains,

$$\omega_s^2 = \frac{c_s^2 k_y^2}{1 + \lambda_{De}^2 k_y^2}$$  \hspace{1cm} (7.3)

In the limit \(1 \ll \lambda_{De}^2 k_y^2\), equation (7.3) gives ion plasma oscillations \(\omega^2 = \omega_{pi}^2\). It is important to note that in the presence of inhomogeneity, a new scale \(\frac{\kappa_n}{k_y}\) is added to the system.

If \(\frac{m_e}{m_i} \ll \left(\frac{\kappa_n}{k_y}\right)^2\), then longitudinal and transverse components of electric field can couple to generate a low frequency electromagnetic wave. The divergence and curl of (4.1) give, respectively,

$$\partial_t \nabla.(n_0 v_{e1}) = -\frac{e}{m_e} n_0 \nabla.E_1 - \frac{e}{m_e} \nabla n_0 .E_1 - \frac{1}{m_e} (\nabla.\nabla p_{e1})$$  \hspace{1cm} (7.4)

and

$$\partial_t(\nabla \times v_{e1}) + (\kappa_n \times \partial_t v_{e1}) = -\frac{e}{m_e} \kappa_n \times E_1 - \frac{e}{m_e} \nabla \times E_1$$  \hspace{1cm} (7.5)
where $\kappa_n = |\frac{1}{n_0} \frac{d n_0}{d x}|$ and $\nabla n_0 = +\hat{x}|\frac{d n_0}{d x}|$ has been assumed. If initially electric field was purely electrostatic i.e. $\mathbf{E}_1 = -\nabla \varphi_1$, then it will develop a rotating part as well if $\nabla n_0 \times \mathbf{E}_1 \neq 0$, as is indicated by the right hand side (RHS) of (7.5).

The Poisson equation in this case is,

$$
\nabla \cdot \mathbf{E}_1 = 4\pi n_0 e \left( \frac{n_{i1}}{n_0} - \frac{n_{e1}}{n_0} \right)
$$

(7.6)

Using continuity eqs. Of electrons (4.18) and ions (6.2), the above equation can be written as,

$$
\begin{align*}
&ik_y E_{1y} \left[ L_0^2 W_0^2 \omega^2 - L_0^2 W_0^2 \omega_{pi}^2 - \omega_{pe}^2 W_0^2 + v_{te}^2 k_y^2 \left( 1 - \frac{\kappa_n^2}{k_y^2} \right) \right] \\
&= \kappa_n E_{1x} \left[ L_0^2 W_0^2 \omega_{pi}^2 + \omega_{pe}^2 \left\{ W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 \left( 1 - \frac{\kappa_n^2}{k_y^2} \right) \right\} \right]
\end{align*}
$$

(7.7)
Then (4.16) and (7.7) yield,

\[ a \left[ L_0^2 W_0^2 - \omega_{pe}^2 (W_0^2 + v_{te}^2 k_y^2) (1 - \kappa_n^2/k_y^2) \right] \]

\[ = -\frac{\kappa_n^2}{k_y^2} \left[ \omega_{pe}^2 \left\{ W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 (1 - \kappa_n^2/k_y^2) \right\} \right] \]

\[ + \frac{L_0^2 W_0^2}{\omega^2 \omega_{pi}^2} \alpha \left( a - \frac{\kappa_n^2}{k_y^2} \right) \]

(7.8)

In the limit \( \omega^2, \omega_{pi}^2 \ll \omega_{pe}^2 \), (7.8) gives a linear dispersion relation [Ref.[12], Saleem, Phys. Plasmas 16, 082102 (2009)]

\[ \omega^2 = \frac{1}{H_0} \left[ (\lambda_e^2 k_y^2) v_{te}^2 \kappa_n^2 \left( \frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right) + (a - \kappa_n^2/k_y^2) c_s^2 k_y^2 \left\{ \frac{5}{3} - \left( \frac{\kappa_T^2}{k_y^2} + \frac{\kappa_T \kappa_n}{k_y^2} \right) \right\} \right] \]

(7.9)

If ion dynamics is ignored, (7.9) gives the electromagnetic mode discussed for pure electron plasma.
In the electrostatic limit \((E_{1x} = 0)\), (7.9) becomes,

\[
\omega^2 = \frac{1}{H_1} \left[ \nu_{te}^2 \kappa_n^2 \left( \frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right) + c_s^2 k_y^2 \left\{ \frac{5}{3} + \left( \frac{\kappa_T^2}{k_y^2} + \frac{\kappa_T \kappa_n}{k_y^2} \right) \right\} \right] \quad (7.10)
\]

where \(H_1 = \left\{ (1 + \lambda_D^2 k_y^2) + \frac{2}{3} \lambda_D^2 k_y^2 \right\} \). For stationary ions, (7.10) reduces to (4.25).

In electron-ion plasma, the electromagnetic wave dispersion relation in the quasi-neutrality limit can be written as,

\[
\omega^2 = \frac{\lambda_e^2 k_y^2}{(a - \kappa_n^2/k_y^2)} \nu_{te}^2 \kappa_n^2 \left( \frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right) + \frac{5}{2} c_s^2 k_y^2 \quad (7.11)
\]

The wave geometry is shown in Fig. 7.1.
This shows that ion acoustic wave can have electromagnetic part even in the quasi-neutrality limit if temperature perturbation is also considered.
The instability can occur if (4.24) is satisfied along with

\[
\frac{5}{2} c_s^2 k_y^2 < \frac{\lambda_e^2 k_y^2}{(\alpha - \kappa_n^2 / k_y^2)} v_{te}^2 \kappa_n^2
\]  

(7.12)
8. Summary

1. Ion acoustic wave which is treated as an electrostatic wave can become electromagnetic due to density inhomogeneity under certain conditions.
2. Electrostatic and thermal temperature fluctuations can also become the source for electromagnetic waves in unmagnetized plasmas.
3. The description of the so-called Magnetic Electron Drift Vortex (MEDV) mode is incorrect. Such a pure transverse mode cannot exist.
4. However a new low frequency wave may exist in nonuniform electron plasma. But this wave is partially transverse an partially longitudinal.
5. Since this wave has frequency closer to ion acoustic wave, therefore it can couple with it.
6. The thermal temperature fluctuations also modify the dispersion relation of electrostatic ion acoustic wave in inhomogeneous unmagnetized plasmas.
7. The electromagnetic and electrostatic new modes discussed can become unstable if $\frac{2}{3} \kappa_n < \kappa_T$ in an open system where the steady state is assumed to be given and both density and electron temperature gradients can be parallel.
8. If $\nabla p_{e0} = 0$ is used as a condition for steady state then the above modes become stable. However many linear and nonlinear mechanisms can make these perturbations unstable.
References