Nature and Nature's laws lay hid in night. God said, 'Let Newton be!' and all was light.

Alexander Pope
W boson mass in $W \rightarrow e\nu$

Wajid Ali Khan$^1$

$^1$Ph.D Student
National Centre for Physics
Islamabad

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Introduction

- Theoretical Background
- Transverse Mass of W boson
- Plots of Different Quantities
- Reconstructed W boson
- Summary
Transverse Energy and Momentum Definitions

- Transverse Momentum: momentum perpendicular to beam direction: \( p_T^2 = p_x^2 + p_y^2 \)
- Transverse Energy defined as the energy if \( p_z \) is identically 0:
  \[
  E_T \equiv E(p_z = 0) \quad \text{or} \quad E_T^2 = p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_z^2
  \]
- Using \( \frac{p_z}{E} = \tanh y \)
  \[
  \Rightarrow E_T^2 = E^2 - p_z^2
  \]
  \[
  \Rightarrow E_T^2 = E^2 - E^2 \tanh^2 y = E^2(1 - \tanh^2 y) = E^2 \text{sech}^2 y
  \]
  \[
  \Rightarrow E_T = E \text{sech} y
  \]
- How does \( E \) and \( p_z \) change with the boost along beam direction?
  - Using \( \tanh y = \frac{v}{c} = \frac{mc}{E} = \frac{p_z c}{E} = \frac{p_z}{E} \)
  - Also \( p_z = p \cos \theta \Rightarrow \frac{p \cos \theta}{E} = \tanh y \)
  - \( \beta \cos \theta = \tanh y \)
Transverse Mass of W boson Cont’d.

- Invariant mass is defined as:
  \[ M_{1,2}^2 = (P_1 + P_2)^2 = P_1^2 + P_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2) \]

- Now we switch from \((p_x, p_y, p_z, E)\) to \((p_T, y, m, \phi)\)

- Now \(\vec{p}_1 \cdot \vec{p}_2 = p_x p_x + p_y p_y + p_z p_z\)

- Using \(p_z = E_T \sinh y\)
  \[ \Rightarrow p_{z1} = E_{T1} \sinh y_1 \]
  \[ \Rightarrow p_{z2} = E_{T2} \sinh y_2 \]
  \[ \Rightarrow p_{z1} p_{z1} = E_{T1} E_{T2} \sinh y_1 \sinh y_2 \]

- Azimuthal angle is given by \(\tan \phi = \frac{p_y}{p_x}, \sin \phi = \frac{p_y}{p_z} \Rightarrow \frac{p_y}{\sqrt{p_x^2 + p_y^2}} = \frac{p_y}{p_T}\)
  \[ \Rightarrow p_y = p_T \sin \phi \]
  \[ \Rightarrow p_{y1} = p_{T1} \sin \phi_1 \]
  \[ \Rightarrow p_{y2} = p_{T2} \sin \phi_2 \]
  \[ \Rightarrow p_{y1} p_{y2} = p_{T1} p_{T2} \sin \phi_1 \sin \phi_2 \]
Transverse Mass of W boson Cont’d

Also \( \cos \phi = \frac{p_x}{p_z} \Rightarrow \frac{p_x}{\sqrt{p_x^2 + p_y^2}} = \frac{p_x}{p_T} \)

\( \Rightarrow p_x = p_T \cos \phi \)
\( \Rightarrow p_{x1} = p_{T1} \cos \phi_1 \)
\( \Rightarrow p_{x2} = p_{T2} \cos \phi_2 \)

Since \( E_1 = E_{T1} \sinh y_1 \) & \( E_2 = E_{T2} \sinh y_2 \)

Using all these values in

\[ M_{1,2}^2 = (P_1 + P_2)^2 = P_1^2 + P_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2) \]

We have the following:

\[ M_{1,2}^2 = 2(E_{T1} E_{T2} y_2 \cosh y_1 \cosh y_2 - p_{T1} p_{T2} \cos \phi_1 \cos \phi_2 - p_{T1} p_{T2} \sin \phi_1 \sin \phi_2 - E_{T1} E_{T2} \sinh y_1 \sinh y_2) \]
Transverse Mass of W boson Cont’d . . .

- Using $\beta \equiv \frac{p}{E} \Rightarrow \beta_T E_T = p_T$ in above we have
  - $M_{1,2}^2 \approx 2(E_{T1} E_{T2} \cosh \Delta y - E_{T1} E_{T2} \beta_{T1} \beta_{T2} \cos \Delta \phi)$
  - $M_{1,2}^2 \approx 2(E_{T1} E_{T2} \cosh \Delta y - E_{T1} E_{T2} \beta_{T1} \beta_{T2} \cos \Delta \phi)$

With $\beta_{T1} = \beta_{T2} \approx 1$

- We have the following result:

  $M_{1,2}^2 = 2E_{T1} E_{T2} (\cosh \Delta y - \cos \Delta \phi)$

- We note that $\Delta y \rightarrow 0 & \Delta \phi \rightarrow 0$ the mass of the particle "M" $\rightarrow 0$
- Hence we can say that "angles" generate Mass.
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Since we don’t measure the $p_z$ of neutrino, we cannot construct invariant mass of $W$ boson.

What measurements or constraints we have
- Electron’s Four Vector.
- Neutrino 2-D momentum ($p_T$) and $m=0$.

We construct "Transverse Mass $M_T$" by

1. Form "transverse" 4-momentum by ignoring $p_z$ (or set $p_z = 0$)
   \[ p_T \equiv (E_T, \vec{p}_T, 0) \]

2. Form "transverse mass" from these 4-vectors;
   \[ M_{T1,2}^2 = (P_{T1} + P_{T2})^\mu (P_{T1} + P_{T2})_\mu \]
Transverse Mass Cont’d... 

- \( M_{T1,2}^2 = P^\mu P_\mu = (E_T^\nu + E_T^e)^2 - (p_T^\nu + p_T^e)^2 \)

- \( M_{T1,2}^2 = (E_T^e)^2 + (E_T^\nu)^2 + 2E_T^eE_T^\nu - (p_T^e)^2 - (p_T^\nu)^2 - 2 \vec{p}_T^e \cdot \vec{p}_T^\nu \)

- Since \( E_T^\nu = p_T^\nu \) & \( (E_T^e)^2 - (p_T^e)^2 = m_e^2 \rightarrow 0 \)
- we have \( M_{T1,2}^2 = 2E_T^eE_T^\nu - 2p_T^e p_T^\nu \cos \Delta \phi \)

- Since \( m_e^2 \rightarrow 0 \) we have \( M_{T1,2}^2 = 2E_T^eE_T^\nu (1 - \cos \Delta \phi) \)
Transverse Mass Cont’d... 

- $M_{W}^2 = M_{e,\nu}^2 = 2E_{T1}E_{T2}(\cosh \Delta y - \cos \Delta \phi)$
- Constrain $M_{W} = 80$ GeV and $P_{T}(W) = 0$
  - $\cos \Delta \phi = -1$
  - $E_{T}^{e} = E_{T}^{\nu}$
  - From this we have $E_{T}^{e}E_{T}^{\nu}$ VS $\Delta \eta$
  - $E_{T}^{e}E_{T}^{\nu} = \frac{80}{2(\cosh \Delta \eta + 1)}$

The transverse mass is given by $M_{T,e,\nu}^2 = 2E_{T}^{e}E_{T}^{\nu}(1 - \cos \Delta \phi)$

Using $\cos \Delta \phi = -1$ and $E_{T}^{e}E_{T}^{\nu} = \frac{80}{2(\cosh \Delta \eta + 1)}$

We have the "Transverse Mass" $M_{T,e,\nu}^2 = 2\frac{80^2}{(\cosh \Delta \eta + 1)}$

Clearly $M_{T} = M_{W}$ when $\eta_{e} = \eta_{\nu} = 0$
If we know $\eta_e$ we can find $\eta_\nu$. Since $\Delta \eta = \eta_e - \eta_\nu$

Also if $\Delta \phi = 0 \Rightarrow M_T \to 0$

If $\Delta \phi = \pi$ then $M_T^2 = 2.2E_T^eE_T^\nu$

- If $E_T^e \equiv E_T^\nu$ then $M_T^2 = 4E_T^e$
- From this we have $E_T^e \approx \frac{M_T}{2}$

Similarly we have $E_T^\nu \approx \frac{M_T}{2}$
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Transverse Energy of Electron...
Electron 

\begin{align*}
\text{Gen_\eta_etc} & \begin{array}{c}
\text{Entries} \ 1274 \\
\text{Mean} \ 0.01534 \\
\text{RMS} \ 1.617
\end{array}
\end{align*}
Neutrino...
Delta $\eta$ ...
Transverse Momentum of Electron...
Transverse Momentum of Neutrino...
Generator level delta phi elec & $\nu$ 

<table>
<thead>
<tr>
<th>Delta Phi</th>
<th>Entries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5$</td>
<td>0</td>
</tr>
<tr>
<td>$-4$</td>
<td>2</td>
</tr>
<tr>
<td>$-3$</td>
<td>4</td>
</tr>
<tr>
<td>$-2$</td>
<td>6</td>
</tr>
<tr>
<td>$-1$</td>
<td>8</td>
</tr>
<tr>
<td>$0$</td>
<td>10</td>
</tr>
<tr>
<td>$1$</td>
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</tr>
<tr>
<td>$2$</td>
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</tr>
<tr>
<td>$4$</td>
<td>18</td>
</tr>
<tr>
<td>$5$</td>
<td>20</td>
</tr>
</tbody>
</table>

Mean: $-0.03229$  
RMS: $1.995$
Gen cos phi elec and nu...
Gen cosh eta elec and nu...
Gen deltaR Electron and Nuetrino...
Transverse Mass of W boson with $(1 - \cos \Delta \phi)$

- **Entries**: 1210
- **Mean**: 57.47
- **RMS**: 36.83
Transverse Mass of W boson with \((\cos \Delta \phi = -1)\)
Transverse Mass of W boson...

**Gen_Mt_w**

- Entries: 1210
- Mean: 77.36
- RMS: 48.13

Transverse mass of W boson

W → eν
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Total energy in ECAL by Electron...
Reconstructed Electron’s Transverse Momentum...
Reconstructed Electron’s $\phi$ . . .
Reconstructed Electron’s $E_T$...
Reconstructed Electron’s $\eta$...
Reconstructed Electron’s Energy...
Reconstructed $\Delta \phi$ of electron & $\nu$ . . .
Reconstructed $\cos \Delta \phi$ of elect & $\nu \ldots$

![Histogram of $\cos \Delta \phi$](image)

- **Entries**: 840
- **Mean**: -0.4768
- **RMS**: 0.6361
Reconstructed CaloMet $p_T$...
Reconstructed CaloMet $\phi \ldots$
Reconstructed CaloMet $E_T$...
Reconstructed CaloMet $\eta \ldots$
Reconstructed $M_T^W$ with $(1 - \cos \Delta \phi)$...
Reconstructed $M_T^W$ with $(\cosh \Delta \eta - \cos \Delta \phi)$...
Reconstructed $M^W_T$ with $(\cosh \Delta \eta = \cos \Delta \phi = 1)$...
Reconstructed $\cosh \Delta \eta$ between electron & $\nu$ ...
Just to verify that $E_T^\nu$ & $p_T^\nu$ have same value...
Mass of W after Gaussian Fit...
Mass of W after Gaussian Fit, Log scale . . .
Mass of W after Gaussian Fit, Log scale . . .
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Summary

- It is seen that $E_T$ cut on both $E_T$ & $E_\nu$ lower than 30 GeV (28,26,24,22,20) mass peak shifts towards the lower values.
- Also if $E_T$ cut on both $E_T$ & $E_\nu$ above that 30 GeV (32,34,36,38,40,42) mass peak shifts towards the higher values.
- Reason is that in both the cases we pick the wrong combinations of Electrons and Neutrinos.
- Using

$$M^2_{1,2} = 2E_{T1}E_{T2}(\cosh \Delta y - \cos \Delta \phi)$$

We can veto the Pseudo Rapidity of neutrino by constraining the $M_W = 80\text{GeV}$.

- Referance "Transverse Mass and Width of the W boson" by J.smith, W.L.neervan, J.A.M. Vermaseren at Institute of Theorical Studies,State University of New York.
LHC.......Way to the future....

Thanks......