Adiabatic model for dust atoms and molecules

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Introduction

Although the 1% part of universe has 99% plasma, there are few astrophysical problems where plasma physics solutions have been suggested.

Astrophysical plasma coexists with dust particles in many situations.

These particles are charged either negatively or positively depending on their surrounding plasma environments.
Dusty plasmas

✓ electrons + ions
  + small particle of solid matter
✓ A fully or partially ionized plasma.
✓ Highly massive
  \((m_d \sim 10^6 - 10^{18} \, m_p)\)
Highly charged
  \((q \sim 10^3 - 10^4 \, e)\)
✓ Variable Charge

\[ \frac{\partial q}{\partial t} = I(r, q, t) \]
The total charging current

$$I(r, q, t) = I_{ext} + \sum_{\beta=e,i} I_\beta(r, q, t)$$

where $I_\beta$ is the electronic or ionic current and $I_{ext}$ are external currents due to:

- Photoemission by incidence of UV radiation;
- secondary electron emission;
- thermionic emission etc.
Some novel aspects of dust in plasmas (dust atoms & molecules)

We discuss Nonlinear Screening of dust grains in a homogenous fully ionized electron-ion plasma under the following headings:

- Adiabatic Processes
- Thomas-Fermi Model for Dust Atom
- Motion of Particle in a Central Field
- Dust Molecule
The Model

we assume that the electrons and ions are inertialess

\[ \nabla e|\varphi| + \frac{1}{n_e} \nabla P_e = 0 \]

\[ \nabla Z_i e|\varphi| - \frac{1}{n_i} \nabla P_i = 0 \]

Adiabatic Process

\[ PV^\gamma = \text{const} \]

\[ \frac{T^{3/2}}{n} = \text{const}, \quad \frac{P}{n^{5/3}} = \text{const} \]
$P_e$ and $P_i$ can be expressed in term of density as

$$P_e = n_{oe} T_{oe} \left( \frac{n_e}{n_{oe}} \right)^{5/3}; \quad P_i = n_{oi} T_{oi} \left( \frac{n_i}{n_{oi}} \right)^{5/3}$$

Where $n_{o\alpha}$ and $T_{o\alpha}$ are the mean density and temperature of the species (e,i)

we obtain the densities of electrons and ions

$$\frac{n_e}{n_{oe}} = \left( 1 - \frac{2}{5} \frac{e |\phi|}{T_{oe}} \right)^{3/2}; \quad \frac{n_i}{n_{oi}} = \left( 1 + \frac{2}{5} Z_i \frac{e |\phi|}{T_{oi}} \right)^{3/2}$$
To calculate electrostatic potential field, we use the Poisson equation

\[ \nabla^2 \varphi = 4\pi e (Z_i n_i - n_e) \]

In a region far from the dust grain \( e|\varphi|<T_e,T_i \) so that densities in the approximate form become:

\[ n_e = n_{0e} - \frac{3}{5} \frac{e|\varphi|}{T_{0e}} \quad n_i = n_{0i} + \frac{3}{5} \frac{e|\varphi|}{T_{oi}} \]

Debye Potential

\[ |\varphi| = \frac{Z_D e}{r} \exp \left( -\frac{r}{\lambda_D} \right) \]

\( \lambda_D > \Lambda_D^{B_D} \) as

\[ \lambda_D = \left( \sqrt{\frac{5}{3}} \right) \lambda_D^B \]
Adiabatic model and charging process

\[ \frac{n_e}{n_{oe}} = \left( 1 - \frac{2}{5} \frac{e}{T_{oe}} |\varphi| \right)^{3/2} \]

In the vicinity of grain surface, the electrons having less thermal velocities can not penetrate into the potential barrier of the dust grain. The maximum potential field to be

\[ |\varphi|_{Max} = \frac{5}{2} \frac{T_{oe} (ev)}{e^2} \]

On the other hand, the potential field becomes maximum only on the surface of dust grain, i.e.,

\[ |\varphi|_{Max} = \frac{Z}{r_D} e \]
Thus we obtain:

\[ Z_D = 2.5 \frac{T_{oe}}{e^2} r_D \]

An Important relation between the charge number and the temperature, for a given radius of the dust grain.

Using different values of the temperature of electrons and the radius of the grains, we calculated the magnitude of the charge \( Z_D \) from above relation for various plasma environments and found it in good agreement with the values cited in the Mendis table.
How a large number of ions will circumnavigate the dust grain?

If $T_{oe} \approx T_{oi} = T_o$

$$n_i = n_{oi} \left(1 + Z_i \right)^{3/2}$$

If $T_{oe} \neq T_{oi}$ and $T_{oe} > T_{oi}$

$$n_i = n_{oi} \left(1 + Z_i \frac{T_{oe}}{T_{oi}} \right)^{3/2} \approx n_{oi} \left(Z_i \frac{T_{oe}}{T_{oi}} \right)^{3/2}$$

As $T_i < e|\Phi|$, the ions will mostly remain close to the surface.
In a region close to the dust grain surface, $e|\varphi| > Ti$ leads to nonlinear screening associated with the trapped ions population which can be formed around the dust grain.

In the electron-proton plasma, in the vicinity of the grains, we can write the Poisson equation for vanishingly small $n_e$

$$\nabla^2 \Phi = (1 + \Phi)^{3/2}$$

$$\frac{2Z_i e}{5T_i} |\varphi|$$
Introducing a new function $1 + \Phi = F$ we obtain the Thomas-Fermi equation i.e.,

$$\nabla^2 F = F^{3/2}$$

The function $\Phi$ itself will satisfy this equation when the ions temperature is less than that of electrons so that we can neglect unity in comparison with $\Phi$. 
A simple picture of the motion of protons around the dust grain

Using $T_e \sim T_i$ and taking $\Phi \approx 1$ on the r.h.s. which means that the electrons are pushed out of the region and only the protons reside close to the dust grain. Thus

$$\nabla^2 \Phi = 2^{3/2}$$

Which has solution

$$\Phi = \frac{\sqrt{2}}{3} \frac{r^2}{\rho^2}$$

$$\frac{5}{2} \frac{T}{4\pi n_0 e^2}$$
Consequently the protons will have Potential energy

\[ U_i = \frac{1}{2} m_i \omega_{pi}^2 r^2 \]

\[ \omega_{pi}^2 = 2^{3/2} \frac{4\pi n_{oi} e^2}{3m_i} \]

Ion Langmuir frequency

Protons execute oscillations like Harmonic Oscillators

The standard 3-D oscillator solution gives

Energy levels

\[ E_n = (n + \frac{3}{2})\eta\omega \]

Where \( n = 0,1,2... \)
Then the wave function of the normal ground state

\[
\Psi_o(r) \sim e^{-\frac{r^2}{2\rho_o^2}}
\]

Where \(\rho_o\) is the oscillation length of the proton

\[
\rho_o \sim \sqrt{\frac{2\eta}{m_i\omega_o}} 
\sim 10^{-3} \text{ cm}
\]

**Conclusion**

protons are oscillating in the vicinity of dust grain at a distance \((10^{-8} - 10^{-5} \text{ m})\) larger than the dust grain size so the existence of such gigantic atoms is indeed a possibility
Dust Atom
Size of the atom

Effective potential energy

\[ U_{\text{eff}}(r) = U(r) + \frac{L^2}{2m_e r^2} \]

For the electron density

\[ \frac{n_e}{n_{oe}} = \left(1 - \frac{2 Z_D e^2}{5 T_e r} - \frac{L^2}{5 m_e T_e r^2} \right)^{3/2} \]

\[ L = m V_l r_l = \eta l \quad \text{where} \quad l = 1, 2, 3, \ldots \]

\[ \frac{n_e}{n_{oe}} = \sum \left(1 - \frac{2 Z_D e^2}{5 T_e r_l} - \frac{l^2 \eta^2}{5 m_e T_e r_l^2} \right)^{3/2} \]
If all the energy levels are filled with protons, the number of orbits will be the same as the charge number i.e.,

\[ l = Z_D \]

\[
R_{last} = r_{Z_D} - r_D = \frac{Z_D \eta^2}{2 m_e e^2}
\]

Evidently \( r_{last} \ll \lambda_D \)
Dust Molecule

There are many physical systems where the harmonic oscillator solution is applicable. One such system is a diatomic molecule in which the two atoms vibrate approximately harmonically along the line joining the two atoms.

In quantum theory, molecular structure is described by the well known **Born-Oppenheimer approximation**
Two dust atoms in a plasma

Grain a

Grain b

R separation
Potential energy $V(R)$ having general features: the dip in $V(R)$ provides an attractive well that may be able to support bound states, a short-range repulsion, asymptotically becomes zero on the large $R$.

Plot of the potential energy $V(R)$ an distance $R$ between two dust atoms of a molecule.
To investigate this process, we expand $V(R)$ about the equilibrium position $R_o$:

$$V(R) = -V_o(R_o) + \frac{(R-R_o)^2}{2} \left( \frac{\partial^2 V(R)}{\partial R^2} \right)_{R=R_o} + \ldots$$

Where 1st term is the attractive portion which has minimum value $-V_o$ at the average separation $R_o$. 2nd term gives angular frequency of two dust grains.

$$\omega = \left[ \frac{1}{\mu} \left( \frac{\partial^2 V(R)}{\partial R^2} \right)_{R=R_o} \right]^{1/2}$$
Protonic Energy

\[ E_{\text{pro}} = V(R_o) \approx \frac{(\Delta P)^2}{2m_i} \]

\[ \Delta P \sim \frac{\eta}{R_o} \]

\[ E_{\text{pro}} \sim \frac{\eta^2}{2m_i R_o^2} \]

Vibrational energy

\[ E_{\text{vib}} \sim \eta \omega = \left( \frac{2m_i}{M} \right)^{1/2} \frac{Z_{\text{D}}^{2/3} \eta^2}{m_i a_o^2} \]

Rotational energy

\[ E_{\text{Rot}} \sim \frac{\eta^2}{2MR_o^2} = \left( \frac{2m_i}{M} \right)^{1/2} \frac{\eta^2}{m_i a_o^2} \]

Thus, as in the ordinary molecule,

\[ E_{\text{pro}} \gg E_{\text{vib}} \gg E_{\text{rot}} \]
Dust Molecule

$R_0$

Rotation

Dust Molecule
Exchange Energy

Considering the weak interaction between the clouds of two dust grains and taking into account the Coulomb interaction of the protons,

\[ U = -V \frac{\pi e^2 \eta^2 n^2}{2m_i T} \]

For adiabatic case

\[ U_{adia} = -Ve^2 \left( \frac{n}{2} \right)^{4/3} \]

For the parameters \( n = 10^9 \text{cm}^{-3}, T \approx 300K \), we obtain for \(|U| \sim 0.1\text{eV}\)
Further Suggestions

- Stability of Dust Atom
- For Ultra relativistic temperature
  \[ \nabla^2 \phi = \phi^3 \]
- Self Focusing and Crystallization
- Sheath Problem
- Raman spectroscopy
Conclusion

Quantum mechanical, nuclear and chemical behaviors can also be studied in Plasma Physics

This is not the end but Yet to explore new thoughts